Frequency Assignment Heuristics for Area Coverage Problems
Final Report

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Executive Summary

Radiocommunications Agency Agreement

Frequency Assignment Heuristics for Area Coverage Problems

Final Report

In many frequency assignment algorithms, interference is accounted for in the form of binary constraints. For every binary constraint, the signal from a wanted transmitter is related to that from a single interfering transmitter. A minimum channel separation is specified in order to guarantee a desired signal-to-interference ratio. However, very often interference emanates from multiple sources simultaneously. Multiple interference constraints concentrate on groups of transmitters rather than on pairs. This allows for the inclusion of accumulative interference effects in the model. In previous reports, the spectral requirement (or *span*) of problem instances characterised by binary constraints was determined with surprising accuracy. Additionally, work concentrating on area coverage focused on incorporating the effects of multiple interference into binary constraints. While the algorithms developed, such as *constraint strengthening*, are effective in providing 100% area coverage, there is scope to improve the spectral requirements.

In this document we provide a new technique for managing co-channel interference, which is often the most dominant source of multiple interference. A natural formulation of this approach involves the introduction of *non-binary co-channel set* constraints in addition to binary constraints. These constraints manage co-channel assignment of frequencies to transmitters. The motivation for this approach comes from an analysis of the effects of interfering signals when no attention is paid to multiple interference. While only marginally increasing the spectral requirement of single interferer binary constraints, the additional use of non-binary co-channel set constraints offer improvements in area coverage, with inadequately served reception points only failing on a marginal basis. In order to find assignments subject to both binary and non-binary co-channel set constraints, a general purpose constraint solver (called NBS) has been produced. This extends the capabilities of the FASoft package, which was developed for binary constraints. NBS is

sufficiently flexible to be of further use in a variety of frequency assignment problems. A manual for NBS is given in the appendix.

In addition to developing non-binary co-channel set constraints we give a detailed analysis of the spectral requirements for private mobile radio services using the UHF 2 band (450-470 MHz) in the Greater London area. This analysis has been heavily dependent on the algorithms, software and theory developed in previous reports. The rules that the Radiocommunications Agency currently use when making assignments have been formulated into constraints. While some constraints are binary, those which characterise channel loading are of the non-binary co-channel set variety. The constraints are used to both analyse the current operational assignment and provide details on the performance of alternative assignments. The results suggest that there is significant scope to improve the quality of the service provided when the frequency band is realigned. In addition to modelling the problem using the rules adopted by the Radiocommuncations Agency, we offer an alternative analysis whereby multiple interference is considered. The results suggest that mitigating all instances of multiple interference would increase the spectral requirement.

Contents

Part I

Area Coverage & Multiple Interference

1 Introduction

This report covers the work that has been carried out under the research agreement *Frequency Assignment Heuristics for Area Coverage Problems* between 1st October 1999 and 30th September 2000. This part of the report analyses the role of co-channel interference in the *multiple interferer* problem. The investigation is a collaboration between the Universities of Cardiff and Glamorgan and has been funded by the Radiocommunications Agency. The work has been carried out by Dr. Roger Whitaker under the guidance of Mr. Steve Hurley and Prof. Derek Smith.

1.1 The Interference Problem

We begin by considering the general problem of interference in radio networks. Receivers which wish to receive signals from a particular transmitter also receive unwanted signals from other transmitters. These unwanted signals reduce the quality of the reception from the wanted transmitter. In order to ensure that the quality of the reception from the wanted transmitter is adequate, *constraints* can be formulated on the frequencies which can be assigned to transmitters. Such constraints can be generated using a variety of different assumptions about interference.

The *single interferer assumption* considers the interference from a single source. Consequently constraints generated using this assumption are *binary*, that is, they involve only pairs of transmitters. A typical binary constraint might have the form $|f(t_i) - f(t_j)| \geq C$ where $f(t_i)$ is the frequency assigned to transmitter t_i and C is a required frequency separation. Each binary constraint is generated by a serving transmitter and an interfering transmitter. In contrast, under the more general *multiple interferer assumption*, it is assumed that interference originates from multiple sources simultaneously. The constraints generated under this assumption are *non-binary* as they simultaneously involve constraining the assignments made to a number of transmitters.

1.2 Propagation Model

The quality of the received signal can be assessed at a number of *reception test points* in the area covered by a particular network. The measure used is the signal-to-interference ratio (SIR). Coverage at a particular reception test point requires that each of the one or more wanted signals meets the required SIR, denoted by σ. The frequency assignment is then deemed successful if coverage over all of the reception test points is achieved. There is no reason why an accurate propagation model should not be used in this work. However, for evaluation of algorithms and models, a simple inverse power law is sufficient. This simplistic model can be used because our interest here is not in a direct study of propagation. The model we use explains how rapidly an interfering signal attenuates with distance and channel separation. Even though the model may not be totally accurate, it is sufficiently representative of the attenuation pattern.

If reception point r_i, where $i = 1, \ldots, m$, is tuned to transmitter T_k where $k = 1, \ldots, n$ then the signal strength S_i, at r_i is assumed to be:

$$S_i = \frac{P_k}{d_{ik}^{\gamma}}$$

where P_k is the power of the transmitting signal, d_{ik} is the distance between transmitter T_k and the receiver r_i. For simplicity P_k is assumed to be the same for all transmitters, and $\gamma = 4$ is used in this model. Under the multiple interference assumption, the total interference I_i at reception point r_i is given by:

$$I_i = \sum_{j=1, j\neq k}^{n} \frac{P_j}{d_{ij}^{\gamma}} \theta$$

where n is the number of transmitters, and for each j, θ is taken as:

$$\theta = 10^{\frac{-\alpha(1+log_2 df)}{10}}$$

if the channel separation, *df*, between the wanted and unwanted transmitter is non-zero. Otherwise, i.e. in the case of co-channel interference, θ is taken as 1. The value α is the attenuation factor for adjacent channel interference and is measured in dB/octave. For our purposes α is taken to be 15. This model is that advocated by Leese and Gower [5]. However, this could easily be replaced by a more accurate propagation model. Note that as an alternative to a propagation model, a list of the values of θ for different values of *df* could be used.

1.3 Models for Multiple Interference

In this report we focus on the role of the simplest type of non-binary constraint in mitigating multiple interference, the co-channel non-binary constraint. Throughout we are interested in sets of transmitters which can be assigned the same frequency. There is evidence to suggest that co-channel interference is of most significance in multiple interference problems and some authors have adopted the restriction to co-channel multiple interference as an assumption: for example see Wu and Wey [12]. In Section 2 we begin by explaining how co-channel sets of transmitters can be managed to reduce the effects of multiple interference.

Techniques to find lower bounds on the minimum number of frequencies required to satisfy problems characterised by binary constraints have been successfully developed: see [7, 1, 8, 9]. However, corresponding techniques for problems characterised by non-binary constraints are needed. In Section 3 we give a novel lower bounding technique which can be applied to the multiple interferer problem without pre-generation of constraints. An evaluation and comparison of its performance is given. In Section 4 we evaluate the use of non-binary co-channel constraints in mitigating multiple interference for a number of test problems, using a flexible frequency assignment tool (NBS) developed during the course of this project. This tool is capable of making assignments subject to both binary and/or non-binary constraints. A manual for NBS is included in the Appendix (Section 6).

2 The Role of Co-channel Interference

2.1 Motivation

We begin by highlighting two important points concerning the simple propagation model. The channel separation between the wanted and unwanted signals (parameter *df* in the propagation model) and the relative distances of the unwanted and wanted transmitters from the reception test point are central to the quality of the SIR attained at the reception test point. To demonstrate this, let us consider a simple network with two transmitters, t_1, t_2 and one reception test point r. We assume that the wanted signal is that from t_1 and that the interfering signal originates from transmitter t_2. Let us also assume that the t_1 and t_2 are equi-distant from r and that t_1 and t_2 have equal power.

Consider first the channel separation. In our example, when $df = 0$ the wanted signal and unwanted signal are indistinguishable, having equal power. When $df = 1$, and t_1 and t_2 have equal power then the propagation model from Section 1.2 gives:

$$\frac{I_1}{S_1} = \theta = 10^{\frac{-\alpha(1+log_2 df)}{10}} = \frac{1}{32}.$$

As *df* increases to 2 channels separation the power of the unwanted signal is assumed to reduce to 1/1000 of the wanted signal. When *df* denotes 3 channels separation, the effects of the unwanted signal at the reception point become negligible, with the power of the unwanted signal taken as just 1/7541 of the wanted signal. This shows that the problem of interference rapidly disappears as channel separation between the wanted and unwanted transmitters increases. In this report we focus on managing the potentially most dominant multiple interference, which is caused by the unwanted transmitters which are co-channel with the wanted transmitter.

The ratio of the distance between a reception point and the wanted and unwanted transmitters is important and worth noting. Let us consider the effect of distance between an unwanted transmitter (t_2) and the wanted transmitter (t_1) where t_1 and t_2 are co-channel. When t_1 and t_2 are at equal distance from the reception point then the wanted signal strength and unwanted signal strength are taken to be equal. If the distance of the unwanted transmitter from r doubles to 2 units and γ is taken to be 4, the power of the unwanted signal drops to 1/16 of the wanted signal. When the unwanted transmitter is 3 units away from r the power of the unwanted signal drops to 1/81 of the wanted signal. This shows that the effect of a co-channel interfering signal becomes negligible as distance from the reception point increases.

For each reception point, our strategy is to ensure that the interferers which are co-channel with the wanted transmitter are distributed so that they do not lead to an inadequate SIR. This can be achieved by introducing non-binary constraints which specify subsets of transmitters such that from each subset, not all the transmitters can be given the same channel. This type of constraint can be used in addition to the binary constraints which are used to model the single interferer problem. It is hoped that this extension to the binary constraint (single interferer) model will give good area coverage results when it is assessed using the multiple interferer model.

2.2 Analysis of Assignments made with the Single Interferer Assumption

A natural starting point in dealing with multiple interference is to assess the quality of assignments made using the single interferer assumption. The 95 transmitter instance originating from the problem generator [4] has been used as an illustrative test problem. Binary constraints have been generated to ensure a 9dB SIR at each of 541 reception test points, under the single interferer assumption. An assignment which violates no constraints has then been found using the FASoft package [6]. This assignment has been assessed with respect to area coverage under the multiple interferer assumption in a bid to determine the dominance of co-channel interferers. The distribution of inadequately served reception points is shown in Figure 1. The reception test points which do not have an SIR of at least 9dB under the multiple interferer assumption are highlighted with a red cross. In Figure 2 we analyse the origin of co-channel interference on a typical failed reception test point. The distribution of dominant co-channel interferers is worth noting. The binary constraints generated under the single interferer assumption ensure that in the locality of the reception point, there are no significant co-channel interferers. Also, in the far distance, any co-channel interferer will have little impact on the reception point. Figure 2 emphasises that the problem of managing co-channel interference is one of finding the assignments which manage the

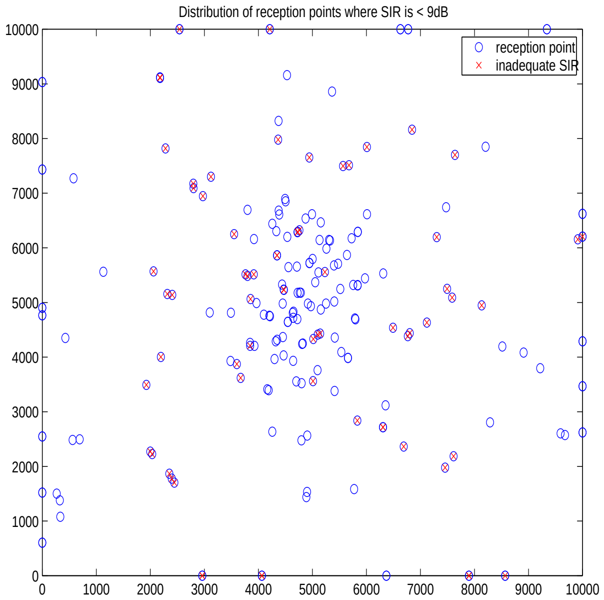


Figure 1: A single interferer assignment assessed with respect to multiple interference

distribution of co-channel interferers in a region which is between the locality of the reception point and the far distance. For each reception point which has an inadequate SIR, analysis of the source of total interference is revealing. In Figure 3 we give a sample analysis for a number of typical inadequately served reception points. These reception points belong to the 95 transmitter problem instance, and are assigned frequencies to ensure a 9dB SIR at each reception point under the single interferer assumption. For each reception point, the transmitters causing at least 5% of total interference have been identified. There is a clear pattern which is easily observed. For each reception point, the interferers identified are either co-channel or first adjacent channel with respect to the wanted transmitter. Also, the most dominant interference originates collectively from co-channel interferers. This suggests that generating (non-binary) constraints to restrict co-channel interference at each reception point may be an effective approach.

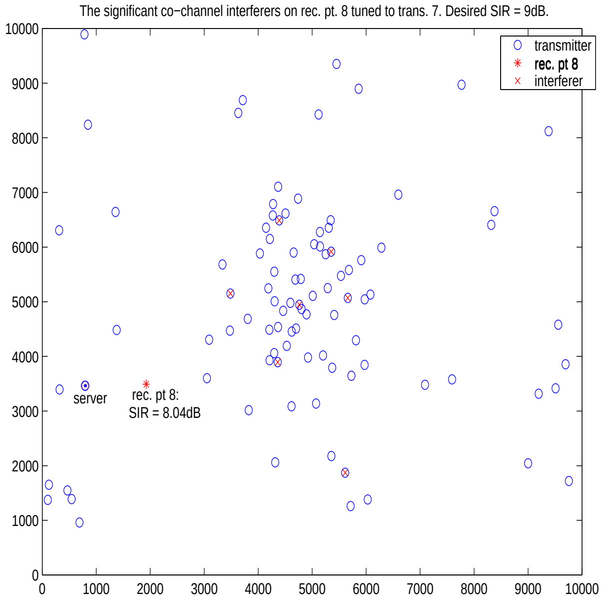


Figure 2: Location of the dominant co-channel interferers (at least 5% of total interference)

```
***************************************************************
rec.  pt 9 tuned to trans 7 on frequency 3:  SIR: 6.39 dB
trans 2 causes 6.82% of intf. Freq sep from trans 7 is 1
trans 14 causes 19.35% of intf. Freq sep from trans 7 is 0
trans 38 causes 35.43% of intf. Freq sep from trans 7 is 0
trans 62 causes 19.21% of intf. Freq sep from trans 7 is 0
**************************************************************
rec.  pt 9 tuned to trans 12 on frequency 5:  SIR: 5.42 dB
trans 2 causes 5.45% of intf. Freq sep from trans 12 is -1
trans 5 causes 10.86% of intf. Freq sep from trans 12 is 1
trans 21 causes 9.74% of intf. Freq sep from trans 12 is 0
trans 51 causes 21.85% of intf. Freq sep from trans 12 is 0
trans 53 causes 9.80% of intf. Freq sep from trans 12 is 0
trans 72 causes 13.10% of intf. Freq sep from trans 12 is 0
trans 76 causes 8.62% of intf. Freq sep from trans 12 is 0
**************************************************************
```

Figure 3: Profile of interferers at typical failed reception points in the 95 transmitter network

3 Lower Bounds and the Multiple Interferer Model

For a given problem, the *span* of an assignment is the difference between the highest and lowest frequencies used. Estimates of the minimum span of an assignment are useful for spectral planning purposes. Many successful techniques have been developed to find useful lower bounds on the minimum span for problems characterised by binary constraints generated by the single interferer model. It is important to note that for any problem instance, since the multiple interferer problem generalises the single interferer problem, the constraints generated under the single interferer assumption are a subset of the constraints generated under the multiple interferer assumption. This was first noticed in [11] and means that lower bounds on the minimum span when adopting a single interferer assumption are lower bounds on the minimum span when the multiple interferer assumption is adopted (although the converse is not true). This is formalised in the following observation.

Observation *If the signal-to-interference ratio with respect to multiple interference is satisfactory at all reception points, then the binary constraints generated using the single interferer assumption still apply (i.e. must be satisfied), and so the lower bounds that can be found using these binary constraints still apply.*

However, it is possible that higher lower bounds on the minimum span could be found for the multiple interferer case, although as yet few such results have been found. Co-channel sets of transmitters can be used directly to obtain a bound for multiple interferer problems, as shown in the following lower bound.

Theorem *Let T be the set of transmitters and let* $\max(t)$ *denote the size of the largest subset of T which can be assigned the same channel as transmitter t (inclusive of t) without causing an inadequate SIR at some reception point. Then the minimum span* (spn) *satisfies:*

$$\text{spn} \geq \lceil \sum_{t \in T} \frac{1}{\max(t)} - 1 \rceil$$

Proof: Assume that a minimum span assignment is made. Let $f(t)$ be the channel assigned to transmitter t and let $n(t)$ be the number of transmitters assigned with channel $f(t)$. Then:

$$\text{spn} + 1 = \sum_{t \in T} \frac{1}{n(t)}.$$

As $\forall t,\ n(t) \leq \max(t)$ it follows that:

$$\text{spn} + 1 \geq \sum_{t \in T} \frac{1}{\max(t)}$$

as was sought.

An interesting feature of this bound is that it does not necessarily require the pre-generation of constraints, only the determination of $\max(t)$. However, the determination of $\max(t)$ for all transmitters t is a non-trivial problem being computationally intensive. Progress has been made by adapting the algorithm given in [2] which is designed to find the maximum clique size. This has involved replacing the sub-routine used to check for cliques to check for valid co-channel sets of transmitters, that is sets of transmitters which can use the same channel without causing any reception point to have an inadequate SIR. Using this approach, the above bound has been implemented on a number of test problems from the problem generator described in [4]. The results are displayed in Figure 4.

The binary lower bound in Figure 4 is derived from the constraint graph associated with the binary constraints generated for the single interferer assumption. The simplest bound is the clique bound [1, 8, 9]. A clique in a graph G is defined as a maximal complete subgraph of G.

Proposition 1 *If C_w is a clique in the subgraph G_w of G with all edges of weight at least w then*

$$\text{spn}(G) \;\geq\; w(|V(C_w)| - 1).$$

where $V(C_w)$ denotes the vertex set of the clique C_w.

Proof: An assignment requires at least $|V(C_p)|$ distinct frequencies and for any two vertices v_i, v_j in C_p we have $|f(v_i) - f(v_j)| \geq w$.

Sometimes tighter bounds are obtained from the travelling salesman bound [8, 9]. Let $H(G)$ denote the smallest sum of edge labels c_{ij} in any Hamiltonian path in G.

Proposition 2 *For a given graph G:*

$$\mathrm{spn}(G) \;\geq\; H(G).$$

Proof: For any chosen minimal span assignment f of G, number of vertices of G as $v_0, v_1, \ldots, v_{|V(G)|-1}$ in ascending order of the channel assigned to them (and arbitrary order for the vertices assigned the same channel). Then $v_0, v_1, \ldots, v_{|V(G)|-1}$ is a Hamiltonian path H_f in G. The span of the assignment is the difference between the largest and the smallest channel used, i.e.

$$\begin{aligned}
\mathrm{spn}(G) &= f(v_{|V(G)|-1}) - f(v_0) \\
&= \sum_{j=0}^{|V(G)|-2} f(v_{j+1}) - f(v_j) \\
&\geq \sum_{j=0}^{|V(G)|-2} c_{v_{j+1}v_j} \\
&= \text{ the sum of the edge labels of } H_f \\
&\geq H(G).
\end{aligned}$$

The binary lower bounds presented in Figure 4 are the best found using Propositions 1 and 2.

Problem Size (No. Trans)	Reqd SIR (dB)	Binary Lower Bound	Co-Channel Set Bound
15 transmitters	9	9	7
15 transmitters	17	11	10
27 transmitters	9	7	6
27 transmitters	17	13	9
45 transmitters	9	8	5
45 transmitters	17	14	10
95 transmitters	9	11	8
95 transmitters	17	25	18

Figure 4: A comparison of lower bounds.

The algorithm used to obtain the results for the co-channel set bound in Figure 4 is very slow on larger problem instances and also in those problems where a high SIR is required at each reception test point. Although this bound does not improve the results from the best binary lower bound, it can be usefully implemented for problems which are not fully characterised by binary constraints, such as the UHF 2 band realignment problem: see Part II of this report for full details of this.

4 Non-Binary Solver (NBS)

An upper bound on the minimum span required by a problem can be found by making an assignment which violates no constraints. The well known FASoft package [6] can be used to make assignments subject to binary constraints. In order to make assignments subject to both binary and non-binary constraints, a general purpose constraint solver has been developed as a part of this project which we refer to as the Non-Binary Solver (NBS). The authors are aware of no similar commercial package tailored to the needs of the constraint satisfaction problem in radio-communications. This package has a number of capabilities which extend those of the FASoft package. In particular, the solver is capable of solving sets of binary constraints as well as sets of non-binary constraints of the co-channel set variety. Also, weights (or penalties) can be associated with the violation of individual constraints. The meta-heuristic used in the package is the tabu search technique which has been shown particularly effective in solving frequency assignment problems: see [3, 6]. The NBS can executed using either a graphical user interface or via the command line. In the latter case either a batchfile can be given as an argument or input can be given via the keyboard. We begin by explaining the algorithm used in the NBS.

4.1 Algorithm used in NBS

The algorithm adopted for use in the NBS is the tabu search algorithm. The algorithm operates for a fixed number of iterations num_{it}, but may terminate early if an optimal solution is found before num_{it} have been performed.

At the initial iteration, an assignment is read in from a file or randomly generated. At each subsequent iteration, another assignment is generated, with the assignment at iteration i being produced by making a change to the assignment generated at iteration $i - 1$. The sequence of assignments produced over num_{it} iterations represents our exploration of the search space of all possible assignments. The assignments generated are each assessed using an objective (or cost) function which associates a cost value to any given assignment. An assignment which causes no constraint violations has a zero cost. The full format of the cost function is given in Section 6.6.

The way in which the assignment at each iteration i is obtained is important.

For a given frequency assignment, a transmitter t is described as *violating* if it is included in a constraint which is not satisfied. Our approach is to generate the assignment at iteration i by changing the frequency assigned to a violating transmitter in the assignment generated at iteration $i-1$.

In order to facilitate this, at each iteration i we define a *neighbourhood set* as a set of triples $\{(t_1, f_1, c_1), (t_2, f_2, c_2), \ldots, (t_k, f_k, c_k)\}$. For each triple (t_j, f_j, c_j), t_j is chosen randomly as a violating transmitter in the assignment from iteration $i-1$. Then c_j is the cost of the assignment from iteration $i-1$ when transmitter t_j has been assigned to frequency f_j. A triple (t, f, c) is then chosen from the neighbourhood set such that the cost c is a minimal representative from the list $c_1, c_2, \ldots, c_k$.

The triple (t, f, c) is then used to produce the assignment at iteration i. If transmitter t has not been assigned to frequency f in the previous R iterations, then the assignment at iteration i is generated by taking the assignment at iteration $i-1$ with transmitter t assigned to frequency f. If t has been re-assigned to frequency f in the previous R iterations, the assignment at iteration i is the same as that produced at iteration $i-1$, unless assigning f to t provides an assignment which has cost lower than any encountered at any previous iteration. This is referred to as the *aspiration criterion.* The parameter R is the *recency list length*, sometimes referred to as *short term memory.* This is used to prevent cycling over a subset of assignments in the search space.

As the search progresses over the num_{it} iterations, the least cost assignment is recorded and updated. The algorithm will only terminate early when a zero cost assignment is found.

In our implementation of the tabu search, the objective function calculates the cost c in each triple (t, f, c) of the neighbourhood set by using a list $\mathrm{ctr}_{t1}, \mathrm{ctr}_{t2}, \ldots, \mathrm{ctr}_{tn}$ of those constraints involving t. This list is used to quickly identify changes in constraint satisfaction which are required to find the cost c by updating the current cost. In the Appendix (Section 6) we explain how to use the NBS.

4.2 Evaluation of NBS

The NBS has been used to assess the effect of using non-binary co-channel set constraints to mitigate multiple interference. These constraints have been used in addition to binary constraints which are there to mitigate interference under the single interferer assumption. The problem generator [4] has been used to generate problem instances for evaluation. For each problem instance, single interferer (binary) constraints have been generated as have non-binary co-channel set constraints. An assignment has then been found which satisfies these constraints using the NBS. The aim has been to use as few channels as possible when making this assignment. The assignment obtained has then been assessed for area coverage. The results have been compared with an alternative approach, the binary constraint strengthening method introduced in [11].

4.3 Constraint Strengthening

The binary constraint strengthening algorithm takes a set of binary constraints generated under the single interferer assumption and progressively adds and strengthens constraints up until 100% coverage (under the multiple interference assumption) is achieved. The algorithm for 100% coverage is shown in Figure 5.

4.4 Span of Strengthened Assignments

The results of [11] show that constraint strengthening is an effective approach in terms of providing good area coverage results, but 100% coverage is expensive in terms of spectrum. Note that in most of our test problems, the assignments which satisfy strengthened binary constraints have a greater span than those which satisfy non-strengthened binary constraints. This is shown in Figure 6. This is an undesirable feature of the constraint strengthening technique as we wish to provide the required coverage using few frequencies.

However, we note that the assignments which satisfy both the non-binary constraints and the binary (single interferer) constraints usually require the same span, or at most a small increase in span. This means that introducing non-binary co-channel constraints is inexpensive in terms of spectrum. These results are summarised in Figure 7.

To compare binary (single interferer) and non-binary co-channel set constraints with strengthened binary constraints, assignments have been made using the same span. This number has been selected as the smallest span required to obtain an assignment satisfying the (single interferer) binary and non-binary constraints. This span is insufficient to find an assignment which satisfies all of the strengthened binary constraints for each of the test problems. Therefore, to assess an assignment made subject to strengthened constraints, the least cost assignment found after 5000 iterations of the NBS has been selected for comparison.

```
Input the required signal-to-interference ratio (SIR) $\sigma$
Input strengthen, the number of constraints to be strengthened
    at each iteration
Find an assignment using the initial binary constraints
    (using the single interferer model)
Calculate coverage (% reception points with $SIR \geq \sigma$)
WHILE (coverage < 100%) DO
    FOR ($i = 1$ to no. reception points) DO
        calculate the SIR at reception point $i$
        IF ($SIR < \sigma$) THEN
            record SIR deficit at $i$
            identify primary interferer (greatest contributor
                to the violating interference)
            add tuned transmitter and primary interferer to list of
                transmitter pairs that may need constraint strengthening
        END IF
    END FOR
    Sort list of violating pairs in decreasing order of SIR deficit
    Increase $c_{ij}$ by 1 for the first strengthen constraints in the list
    Find an assignment using the revised constraints
    Calculate coverage
END WHILE
Output assignment and span
```

Figure 5: *The constraint strengthening algorithm*

No. trans	Required SIR	Best span before strengthening	Best span after strengthening
15	9dB	9	9
15	17dB	11	12
27	9dB	7	8
27	17dB	15	17
45	9dB	8	9
45	17dB	15	17
95	9dB	11	16
95	17dB	26	35
458	9dB	11	13
458	17dB	19	28

Figure 6: The best spans for binary and strengthened binary constraints.

No. trans	Required SIR	Binary Constraints		Non-Binary Constraints		Both Types:
		Lower Bound	Upper Bound	Lower Bound	Upper Bound	Upper Bound
15	9dB	9	9	7	9	9
15	17dB	11	11	10	11	11
27	9dB	7	7	6	7	7
27	17dB	13	14	9	15	15
45	9dB	8	8	5	8	8
45	17dB	13	15	10	16	16
95	9dB	11	11	8	-	11*
95	17dB	25	26	18	30	31
458	9dB	11	11	-	-	11*
458	17dB	19	19	-	-	20*

(* denotes that the non-binary constraints are of restricted arity).

Figure 7: A comparison of the spans for different types of constraints.

4.5 Form of Non-binary Constraints

It is interesting to note that the form of the non-binary constraints which we are proposing is very much dependent on the required SIR at each reception point. In particular, if the desired SIR is very high, the non-binary constraints have low arity, that is each constraint involves few transmitters. In an extreme case where the SIR is very high indeed, it is possible that binary constraints may suffice.

This phenomenon may be explained by considering a particular reception point. If a very high SIR is required, then only a small total amount of multiple interference can be tolerated. It is likely that this small total amount of interference will be achieved by considering the signals from only a small number of unwanted transmitters which are co-channel with the wanted transmitter. Thus the non-binary constraints which manage the combined effect of this interference at the reception point will involve only a small number of transmitters.

4.6 Area Coverage

In Figures 9 and 10 we display the area coverage results at a required SIR of 9dB and 17dB respectively. Additional information is also given in Figure 8.

Combined binary & non-binary constraints can be used to find assignments that are at least as good, in terms of area coverage results, as the assignments obtained from binary constraints and strengthened binary constraints. Note also that in general, the non-binary constraints are most effective in providing good area coverage results when the SIR is low (9 dB). For the larger problem instances, a restriction has been placed on the maximum arity of the non-binary constraints. This is necessary in order to restrict both the number of constraints produced and the time taken to produce them. The non-binary constraints for the 95 and 458 transmitter problems at a required SIR of 9dB are of arity 3. The non-binary constraints for the 458 transmitter problem at 17dB are of arity 3 and 4. The effect of restricted arity non-binary constraints is disappointing, but this may be due in part to the larger problem size. This suggests that large arity constraints are important and should not be neglected when using the non-binary constraint approach.

The area coverage results only offer a limited metric on the quality of an

No.	Reqd	Coverage (%)			
Trans	SIR (dB)	Binary	Non-Binary	Binary & Non-Binary	Strengthened Binary
15	9	93.06	93.06	97.22	97.02
15	17	84.72	55.56	86.11	86.11
27	9	86.42	91.43	94.29	85.71
27	17	90.71	72.85	93.57	90.00
45	9	87.97	92.53	93.36	91.70
45	17	81.33	61.41	90.46	87.55
95	9	78.37	49.54*	82.44*	69.87
95	17	78.37	72.09	90.20	80.77
458	9	88.00	54.32*	88.22*	80.64
458	17	71.85	32.52*	71.93*	68.41

($*$ denotes that the non-binary constraints are of restricted arity.)

Figure 8: A comparison of area coverage results

assignment. In the following sections we are able to see more clearly the benefits of using non-binary co-channel set constraints.

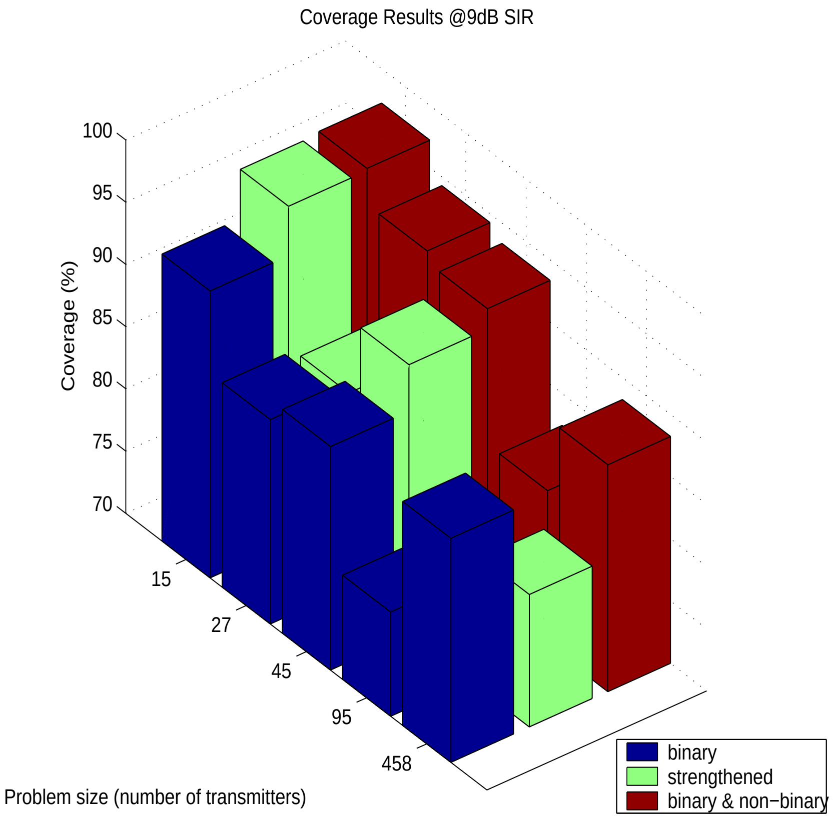


Figure 9: A Comparison of area coverage results at 9dB SIR: Binary constraints (blue), strengthened binary constraints (green) and binary & non-binary constraints (red)

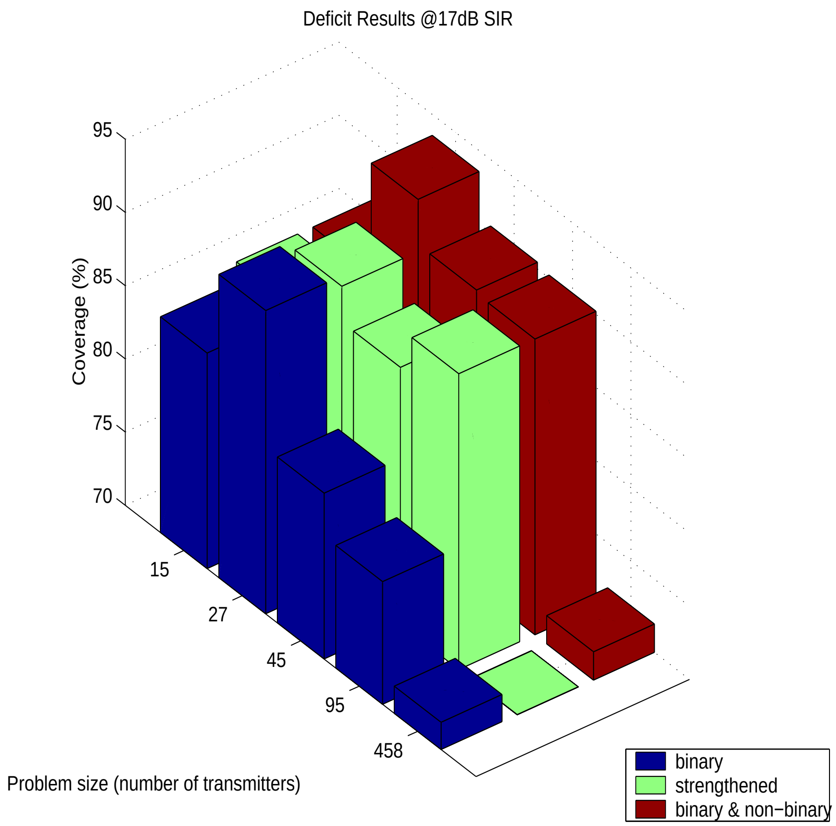


Figure 10: A Comparison of area coverage results at 17dB SIR: Binary constraints (blue), strengthened binary constraints (green) and binary & non-binary constraints (red)

4.7 SIR deficit

For each reception point which receives an inadequate SIR, we define the deficit as the difference between the actual SIR and the required SIR. For example, when an SIR of 9dB at each reception point, a reception point which has an SIR of 6.3dB is said to have a deficit of 1.7. The total of deficits over all inadequately served reception points gives us another useful measure of the quality of an assignment. Clearly a small total SIR deficit is most desirable. A comparison of the total SIR deficits is given in Figures 13 and 14. Additional information is given in Figure 11. A comparison of the average SIR deficits is given in Figures 15 and 16. Additional information is given in Figure 12.

No. Trans	No. Reception Points	Reqd SIR (dB)	Total Deficit			
			Binary	Non-Binary	Binary & Non-Binary	Strengthened Binary
15	72	9	1.81	1.82	1.46	2.30
15	72	17	11.20	81.46	10.82	65.90
27	140	9	25.63	6.51	4.33	66.27
27	140	17	14.63	79.34	9.02	54.07
45	241	9	28.47	14.67	14.11	107.65
45	241	17	112.01	238.85	25.61	195.98
95	541	9	240.35	1856.03*	219.81*	887.11
95	541	17	198.37	363.18	72.96	869.68
458	2675	9	383.68	7585.57*	381.33*	4233.69
458	2675	17	1577.15	14166.89*	1605.98*	8659.73

($*$ denotes that the non-binary constraints are of restricted arity.)

Figure 11: A comparison of total SIR deficit results

No. Trans	No. Reception Points	Reqd SIR (dB)	Average Deficit			
			Binary	Non-Binary	Binary & Non-Binary	Strengthened Binary
15	72	9	0.36	0.36	0.73	1.15
15	72	17	1.02	2.54	1.08	6.59
27	140	9	1.34	0.54	0.54	3.31
27	140	17	1.13	2.09	1.00	3.86
45	241	9	0.98	0.82	0.88	5.38
45	241	17	2.49	2.57	1.11	6.53
95	541	9	2.05	6.80*	2.31*	5.44
95	541	17	1.70	2.41	1.38	8.36
458	2675	9	1.20	6.21*	1.21*	8.17
458	2675	17	2.09	7.84*	2.14*	10.25

(∗ denotes that the non-binary constraints are of restricted arity.)

Figure 12: A comparison of average SIR deficit results

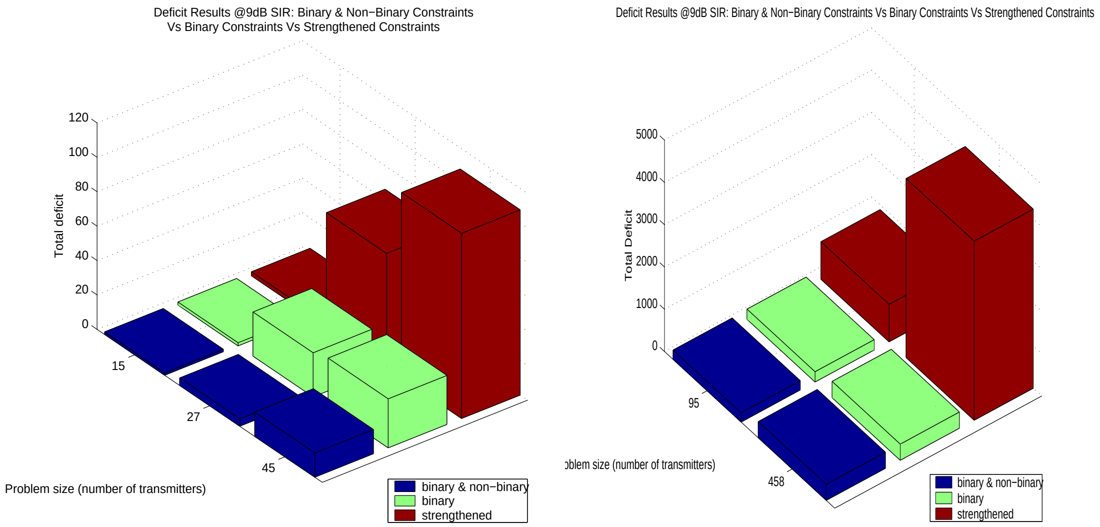


Figure 13: A Comparison of total deficit results at 9dB SIR.

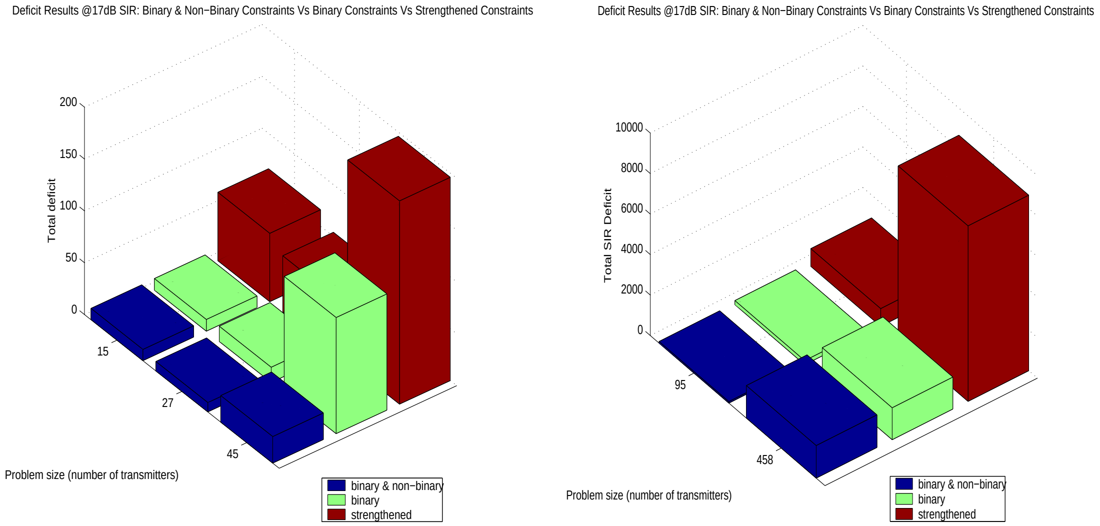


Figure 14: A Comparison of total deficit results at 17dB SIR.

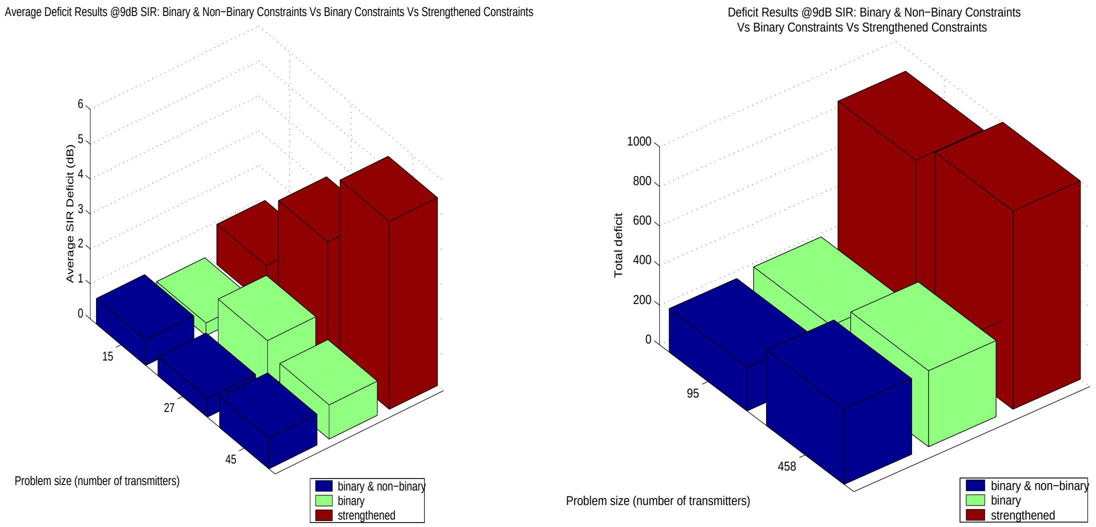


Figure 15: A Comparison of average deficit results at 9dB SIR.

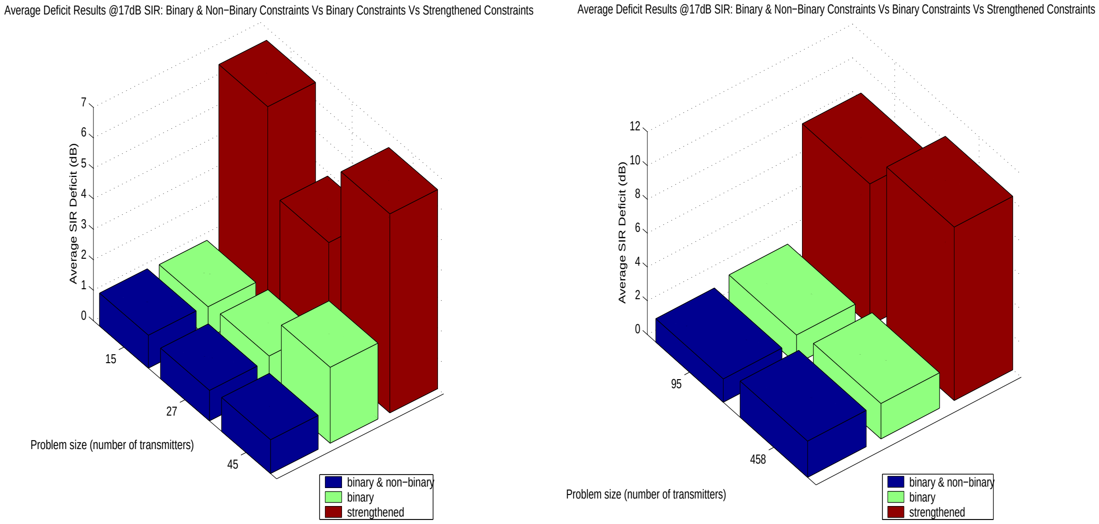


Figure 16: A Comparison of average deficit results at 17dB SIR.

Note that there is a significant difference in the average SIR deficit when different types of constraints are used to find assignments for the same problem instance. Figures 13 and 14 show that the non-binary constraints are effective in reducing the average and total amount of interference incurred at reception points. Consequently those reception points which are inadequately served have only done so on a marginal basis.

The binary constraint strengthening results are not good in terms of SIR deficit. Although the approach works well as a minimum span method, its performance here is poor. This is because we are concerned with a fixed spectrum problem. Therefore the assignments which have been found have violated some of the strengthened constraints. These violations have caused a number of reception points to be very poorly served indeed, causing a large total SIR deficit. Violating constraints causes a significant problem if the constraints involve transmitters which are geographically close together. Then any reception point in such a region is significantly affected.

As an alternative to considering the SIR deficit, it is interesting to consider the worst SIR incurred at any reception point. This is useful as it gives us another measure of how close we are to having 100% coverage. For example, if the worst SIR is 7.1dB in a problem where we require 9dB, then we are able to say that we can provide 100% coverage at 7.1dB or less. Obviously it is desirable to have an assignment where the worst SIR is close to desired SIR.

4.8 Worst reception point SIR

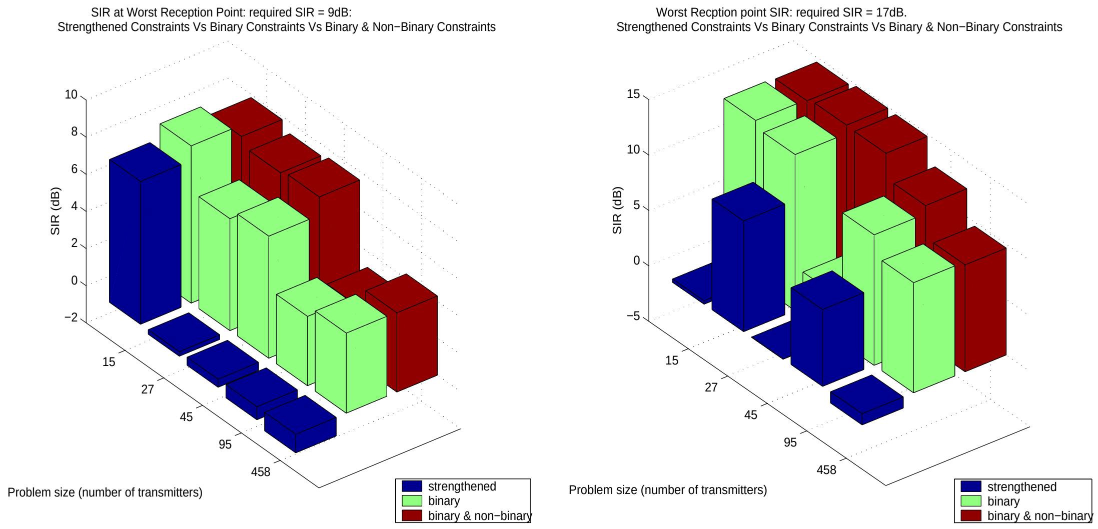
The SIR at the most inadequately served reception point has been found for each of the assignments considered. These are plotted in Figure 18 and additional details are given in Figure 17.

The assignments made subject to the strengthened constraints perform particularly badly. This is due to the violation of some of the constraints, as mentioned when analysing the total SIR deficit. Adding the non-binary constraints to binary constraints gives an improvement over just using the binary constraints.

No. Trans	Reqd SIR (dB)	Worst SIR (dB)			
		Binary	Non-Binary	Binary & Non-Binary	Strengthened Binary
15	9	8.47	8.37	7.84	7.69
15	17	14.68	12.10	14.56	0.30
27	9	6.03	7.64	7.37	−0.24
27	17	14.08	11.69	14.84	9.98
45	9	6.57	7.56	7.55	−0.43
45	17	3.97	11.61	14.77	−0.02
95	9	3.73	−5.34*	3.14*	−0.70
95	17	11.78	11.04	12.52	6.93
458	9	4.31	−3.14*	4.27*	−0.97
458	17	9.93	−3.11*	9.64*	−0.41

(∗ denotes that the non-binary constraints are of restricted arity.)

Figure 17: A comparison of worst SIR results


SIR at Worst Reception Point: required SIR = 9dB:
Strengthened Constraints Vs Binary Constraints Vs Binary & Non−Binary Constraints

Worst Recption point SIR: required SIR = 17dB.
Strengthened Constraints Vs Binary Constraints Vs Binary & Non−Binary Constraints

Figure 18: A Comparison of the quality of signal at the worst reception point: left- 9dB SIR required, right- 17dB SIR required.

5 Conclusions

In this report we have made a preliminary investigation into the role of the (non-binary) co-channel set constraint in mitigating multiple interference. A number of important conclusions can be drawn from our results.

Co-channel set constraints currently represent the most effective method of mitigating multiple interference using a relatively small number of channels. Assignments satisfying both the binary (single interferer) constraints and the non-binary co-channel set constraints can often have a span approximately equal to the best upper bound on the binary minimum span. Assignments satisfying strengthened binary constraints often have a higher span. When solving strengthened binary constraints as a fixed spectrum problem, the violations which are made can cause certain reception points to have a very low SIR indeed. A low SIR is likely at a reception point r when a number of constraint violations are made which involve transmitters local to r.

It is important to note that the results for binary constraints could be interpreted as being an over-estimate and must be compared with the combined binary and non-binary constraints with caution. Note that there is much variation in the coverage provided amongst different solutions to the same set of binary (single interferer) constraints. This is because it is possible that the assignments produced will satisfy some of the non-binary constraints on an *ad-hoc* or random basis. Assignments from binary constraints which perform well in terms of SIR quality will also inadvertently satisfy many of the non-binary constraints. However, this element of chance can be removed by the explicit inclusion of non-binary co-channel set constraints, which will guarantee that the co-channel interference in isolation will not lead to inadequate SIR.

Co-channel set constraints are most effective when the required SIR is low. This is because, under these conditions, adjacent channel interference has less effect on whether the desired SIR will be achieved, and so co-channel interferers are of greater importance. The benefits of using non-binary co-channel set constraints can be seen most clearly in terms of the total SIR deficit associated with the resultant assignment. The non-binary constraints have the effect of distributing channels such that the majority of inadequate reception point only fail on a marginal basis. However, our results indicate that in order to further improve the quality of assignment, there is a need to manage the combined effect of co-channel and (at least) first adjacent channel interference. Including first adjacent channel interference in the formulation of non-binary constraints is a non-trivial problem, and may increase the minimum span of assignments.

An important issue concerns the arity of non-binary constraints. Restricted, small arity constraints have the advantage of being quick to compute for large problems. Explicitly generating all co-channel set constraints for large problems is not a feasible option due to computational complexity. Unfortunately, it appears that low arity constraints have little effect in the larger problems where the re-use of frequencies is relatively high, such as in the 458 transmitter problem. A fast method of producing the most important subset(s) of non-binary constraints for larger problems would be a breakthrough, making larger problem instances tractable.

6 Appendix: Using NBS

In this section we give instructions on how to use the NBS software. The software can be operated in three different ways. For windows operating systems, a user interface can be used to input the parameters. For UNIX systems, NBS can be operated interactively or remotely from the command line, by specifying the name of a batchfile as an argument to the executable.

6.1 Constraints

The NBS will solve binary constraints and/or non-binary constraints. Constraints must be read in from two files, one containing the binary constraints and one containing the non-binary constraints. Only one file is required if just one type of constraint is involved.

6.1.1 Binary Constraint File Format

Binary constraints should be in the "CELAR" format as used in the FASoft package. Each line of the binary constraint file should be of the format:

$$i\,j\ >\ k$$

where i and j are transmitters between which more than k channels separation are required. Every line in the file should end with a carriage return. The exact positioning of $i, j, >, k$ along the line is unimportant provided there is at least one blank space between adjacent entries. For example:

$$4\,7\ >\ 3$$

would constitute a valid line in a binary constraint file, indicating that transmitters 4 and 7 require frequencies with a separation of strictly greater than 3. An example of a binary constraints file is given in Figure 19.

6.1.2 Non-Binary Constraint File Format

Each line of the non-binary constraint file should be of the following format:

```
0 1 > 0
0 2 > 0
0 19 > 0
0 31 > 0
1 2 > 0
1 4 > 0
1 31 > 0
2 3 > 0
2 4 > 0
2 5 > 0
2 12 > 0
2 19 > 0
2 31 > 0
```

Figure 19: An extract from a binary constraint file.

$i\ j\ k \ldots r$

where $j, k, \ldots, r$ are transmitters which cannot all be co-channel and i is the size of this set. Every line in the file should end with a carriage return. The exact positioning of $i, j, k, \ldots, r$ along the line is important. i should be the first entry along the row and exactly one blank character should separate subsequent entries. The last entry, r, along the row should be immediately followed by a carriage return. For example:

5 12 4 7 9 15

would constitute a valid line in a non-binary constraint file, indicating that the 5 transmitters 12,4,7,9,15 are not permitted to be assigned to the same channel. Note that it is only necessary to include the *minimal* co-channel sets. For example, if the above constraint is included, then there is no need to include the constraint:

7 12 4 7 9 15 33 23

as we have already specified that 12,4,7,9,15 are not permitted to be co-channel. An example of a non-binary constraint file is given in Figure 20.

```
2 0 7
2 0 9
2 0 12
2 0 13
2 0 14
3 0 2 5
3 0 2 8
4 1 2 10 11
3 0 5 8
4 2 5 10 11
3 2 8 10
3 2 8 11
3 3 10 11
```

Figure 20: An extract from a non-binary constraint file.

6.1.3 Naming Conventions

The constraint files can be named in any way. However, the following conventions are recommended as they have been extended from the FASoft package.

CONSTRAINT FILE	NAME
binary	*filename.ctr*
non-binary	*filename.ctr.nb*

6.2 Constraint Weighting Files and Naming Conventions

In many constraint satisfaction problems, each constraint can be given a weight (or penalty) which reflects the importance of its satisfaction. These can be included as an optional input to the NBS. For each variety of constraint (ie binary or non-binary), the associated weights can be given in a file called the constraint weighting file. Each line of a constraint weighting file should contain only a single integer, followed immediately by carriage return. The integer should be the first entry on the line. For a particular variety of constraint, the entry on the i^{th} row of a constraint weighting file identifies the weight of the i^{th} constraint listed in the constraint file. Note for each type of constraint, the constraint weighting file and the constraint file should have the same number of rows. An example of a weighting file for the constraints in Figure 20 is given in Figure 21.

If constraint weighting files are not included as input to the NBS, then the program will automatically set all constraints to have an equal weighting of 1.

```
1
2
1
1
1
4
1
3
3
6
1
1
1
```

Figure 21: A weighting file for the constraints in Figure 20.

If constraint weighting files are to be included, then any file name can be given. However, the following conventions are recommended.

WEIGHTING FILE	NAME
binary	*filename.ctr.wt*
non-binary	*filename.ctr.nb.wt*

6.3 Cost Scalar Values

During the search carried out by the NBS, assignments are assessed using a cost function. This cost function is the sum of the cost derived from binary constraint violations and the cost derived from non-binary constraint violations. Non-negative integers w_b, w_{nb}, so called *cost scalar values* can be entered to scale the cost from binary constraint violations and from non-binary constraint violations in the cost function. The total cost of an assignment is composed as:

$$w_b \times \text{binary cost} + w_{nb} \times \text{non-binary cost}$$

The detailed format of the cost function is fully explained in Section 6.6.

6.4 Operational Files

The NBS requires a number of files for operational purposes, such as to start the search and to record its progress.

6.4.1 Start File

The NBS can either commence searching with a randomly generated assignment or after reading in an assignment from a file, known as a start file. The start file is optional. If no start file is declared, the NBS automatically generates a random assignment. Otherwise the assignment in the start file should be of the list format style as used in FASoft. The format for such a file is strict and the reader is referred to the FASoft user guide for full details. We note that using this format, we can "fix" the assignment made to a transmitter: that is ensure that is remains unchanged during the NBS search. It is recommended that the start file be named as *filename.s*. An example of an assignment is given in Figure 22.

6.4.2 Domain File

The NBS needs to know which frequency values to use during the searching process. This can be done in two ways. Either the number of frequencies needs to be declared or alternatively the name of a domain file should be specified. In the former case, the number specified should be a positive integer, where upon the assignment for each transmitter will be selected from the frequency domain:

$$1, 2, 3, \ldots, n-1, n.$$

If the name of a file is given then it should have the same format as a domain file for FASoft. The reader is referred to the FASoft user guide [10] for full

```
trans frequency fix
    0         2   0
    1         4
    2        11
    3         8   0
    4        12
    5         2
    6        10
    7         6
    8         9
    9         1   0
   10        11   0
   11         4
   12         7
   13         3   0
   14         5
```

Figure 22: An example of a start file and an assignment file.

details. It is recommended that the domain file be named as *filename.dom*. In Figure 23 we give an example of a domain file. Each row of the domain file corresponds to a domain. Each domain is labelled according to the first integer along each row. The second integer along the row is the size of the domain. Subsequent integers along the row are frequencies in the domain. Each row must end with a carriage return. Unlike FASoft, the initial domain need not be the union of all other domains.

```
0 5 2 3 4 7 8
1 3 2 7 8
2 2 4 8
3 2 2 4
```

Figure 23: An example of a domain file.

6.4.3 Var File

The var file tells the NBS which frequency domains each transmitter is permitted to access. The var file should have exactly the same format as FASoft: see [6], but should only include the first and second columns. The subsequent optional columns are not required. An example of a var file is given in Figure 24. The entries in the first column are the transmitters while the entries in the second column are the associated domains. Any transmitter absent from the domain file will be allocated to the zero domain. It is recommended that the var file be named as: *filename.ctr.var*. As an alternative to specifying a var file, it is acceptable to enter the number of transmitters involved in the problem. When this occurs, the NBS will associate all transmitters with the zero domain. Note also that the NBS will assume that the transmitters are indexed from zero.

```
2 1
3 0
8 1
9 2
10 2
```

Figure 24: An example of a var file.

6.4.4 Assignment File

The assignment file is used to record the best assignment found during the course of the search. The assignment is given in the FASoft list file format. It is recommended that the assignment file be named as *filename.f*. An example of an assignment file is given in Figure 22.

6.4.5 Log File

The log file is used to record the progress of the search, and gives information on constraint violation and costs of the current assignment and best assignment found. It is recommended that the log file be named as *filename.log*. An example of a log file is given in Figure 25.

```
**** ITERATION: 1430 ****
*** BINARY CONSTRAINTS: 2298 of these ****
Current number of violations:  95
Violations (binary constraints) in best assignment:  87
Current cost from binary constraints:  190
Cost (from binary constraints) in best assignment so far:  174
*** NON-BINARY CONSTRAINTS: 642200 of these ****
Current number of violations:  170
Violations (non-binary constraints) in best assignment:  165
Current cost from non-binary constraints:  408
Cost (from non-binary constraints) in best assignment so far:
368
*********************************************
Current total violations:  265
Current total cost:  592
Total violations from best assignment:  252
Total cost from best assignment:  542
```

Figure 25: Output included in the log file

6.5 Search Parameters

A number of search parameters can be adjusted by the user. These control the length of search, amount of short term memory used by the tabu search, the size of the random violating neighbourhood and the format of the cost function.

6.5.1 Iterations

The iterations parameter controls the number of iterations the search will perform. The default value is set at 5000.

6.5.2 Recency List

This controls the amount of short term memory used by the tabu search. This has a default setting of 6% of the number of transmitters.

6.5.3 Neighbourhood Size

The tabu search uses a random violating neighbourhood, where the default size is set as 25% of the number of transmitters.

6.5.4 Binary Cost Power

The binary cost power parameter is only required when binary constraints are included in the problem. A non-negative integer must be supplied, usually 0,1 or 2. The role of this parameter is explained in the following Section.

6.6 Cost Function

The NBS uses a cost function to assess potential assignments as to their suitability. This cost function is calculated 'per transmitter'. The cost per transmitter is composed of a cost derived from binary constraint violations and a cost derived from non-binary constraint violations. We use the following notation in our explanation.

w_b :	binary cost scalar.
w_{nb} :	non-binary cost scalar.
bin-viol(t,i) :	amount of violation at i^{th} binary constraint involving transmitter t.
bin-viol$^*(t,i)$:	binary (0-1) variable which is 1 if and only if the i^{th} binary constraint involving t is violated.
nonbin-viol(t,i) :	binary (0-1) variable which is 1 if and only if the i^{th} non-binary constraint involving t is violated.
bin-wt(t,i) :	weight of i^{th} binary constraint involving transmitter t.
nonbin-wt(t,i) :	weight of i^{th} non-binary constraint involving transmitter t.
p :	binary cost power (non-negative integer).

The cost for transmitter t derived from binary constraints (denoted bin-cost(t)) is:

$$\text{bin-cost}(t) = w_b \sum_i \text{bin-wt}(t,i) \times \text{bin-viol}(t,i)^p \text{ if } p \geq 1$$

else

$$\text{bin-cost}(t) = w_b \sum_i \text{bin-wt}(t,i) \times \text{bin-viol}^*(t,i) \text{ if } p = 0.$$

The cost for transmitter t derived from non-binary constraints (denoted nonbin-cost(t)) is:

$$\text{nonbin-cost(t)} = w_{nb} \sum_i \text{nonbin-wt}(t,i) \times \text{nonbin-viol}(t,i).$$

Then the total cost of an assignment (TC) is:

$$TC = \sum_t \text{bin-cost}(t) + \text{nonbin-cost}(t).$$

It is important to understand the role of the binary cost power variable. If this variable is set to zero then cost at transmitter t from binary constraints is dependent on the number of binary constraint violations involving t. When the binary cost power variable is set to 1, then bin-viol$(t,i)^p$ records the amount by which the i^{th} constraint involving transmitter t violated. Larger constraint violations can be penalised more than smaller ones on setting p as 2 (or more).

6.7 Batchfile Input

The NBS can be operated from the command line without using the graphical user interface. The executable is named *solver.bin* (for UNIX systems) and *solver.exe* for windows systems. The program will ask for each of the parameters described in the previous section. Optional parameters can be omitted by entering a dash – when asked to specify an input.

The solver will also read the input parameters from a file, which we refer to as a *batchfile*. This is achieved by giving the name of the batchfile as an argument to the executable. The format of the batchfile is strict. The file must contain exactly 15 lines, one line for each of the parameters. Each parameter must being of in the first column of each line and be following immediately by a carriage return. The order of parameters in the file should be as in Figure 26.

If an optional parameter is not to be entered or if a default setting is to be selected, then a dash – should be given on the appropriate line. An example of a batchfile is given in Figure 27.

```
binary constraint file path
binary weighting file path
binary cost scalar
non-binary constraint file path
non-binary weighting file path
non-binary cost scalar
start file path
domain file path
var file path
assignment file path
log file path
number of iterations
recency list length
Neighbourhood size
Binary cost power
```

Figure 26: The format of the batchfile

```
95t17db.ctr
-
1
95t17db.ctr.nb
-
1
start.f
30
95
95t17db.f
95t17db.log
15000
-
-
1
```

Figure 27: An example of a batchfile

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Part II

UHF 2 Band Realignment

1 Introduction

The UHF 2 band covers frequencies in the range 450-470 MHz. In order to comply with continental practice the Radiocommunications Agency [1] is to re-allocate the band and generate a new operational assignment. The region being considered is the area within 60 km of Charing Cross station, London. Licenses for this band are mainly issued for private business radio use, and are of two types, *single* and *dual*. Single licenses require the use of a single channel, while dual licenses require a channel pair, with one channel for transmission and one for reception. The receive assignment is interpreted as a copy of the transmit assignment. Licenses are placed into *business classes* dependent on their use.

1.1 Business Classes

The licenses in each business class each have the same *co-channel re-use distance*, that is the distance beyond which the same channel can be re-used without users incurring unacceptable interference. The business classes are of two types- those which use *on-site systems* licenses and those which use *wide area systems* licenses.

- **On-site systems:** Users of on-site system licenses are required to operate no more than 3 km away from their local base station (or site address should they not choose to operate a base station). The on-site type service is the most heavily used in major conurbations, as it is ideally suited to the needs of security companies in shops and factories. In London and a number of other cities this has meant that the number of applicants far exceeds the available spectrum.
- **Wide area systems:** Wide area systems are those licenses which are assigned a much larger coverage area: up to 30 km.

The business classes are:

- O5: on-site, single channel, non-specific industry, 6 km re-use distance, 2220 licenses.

- U3: wide area, single channel, UK general service, 0 km re-use distance (i.e. exclusive channel per license is preferred), 5 licenses.
- RC: on-site, single channel, unspecified use, 0 km re-use distance (i.e. exclusive channel per license is preferred), 6 licenses.
- P4: wide area, single channel, unspecified use, 5 km re-use distance, 1 license.
- T1: wide area, single channel, taxi and car hire, 2 km re-use distance, 49 licenses.
- D5: on-site, dual channel, non-specific industry, 6 km re-use distance, 604 licenses.
- G3: wide area, dual channel, non-specific, general industrial use, 5 km re-use distance, 204 licenses.
- H3: wide area and on-site, dual channel, hospitals, doctors, midwives, 5 km re-use distance, 15 licenses.
- H4: wide-area, dual channel, motor cycle dispatch, 2 km re-use distance, 6 licenses.
- M6: wide-area, dual channel, local government, 5 km re-use distance, 24 licenses.

In total there are 3134 licenses, of which 2226 are single and 908 are dual licenses.

1.2 Transmitter Locations

For each license, the location of the transmitter (base station) at which the channel assignment is to be made has been given by the Agency. These locations have been plotted in Figure 28. Transmitters are most dense around the Charing Cross region.

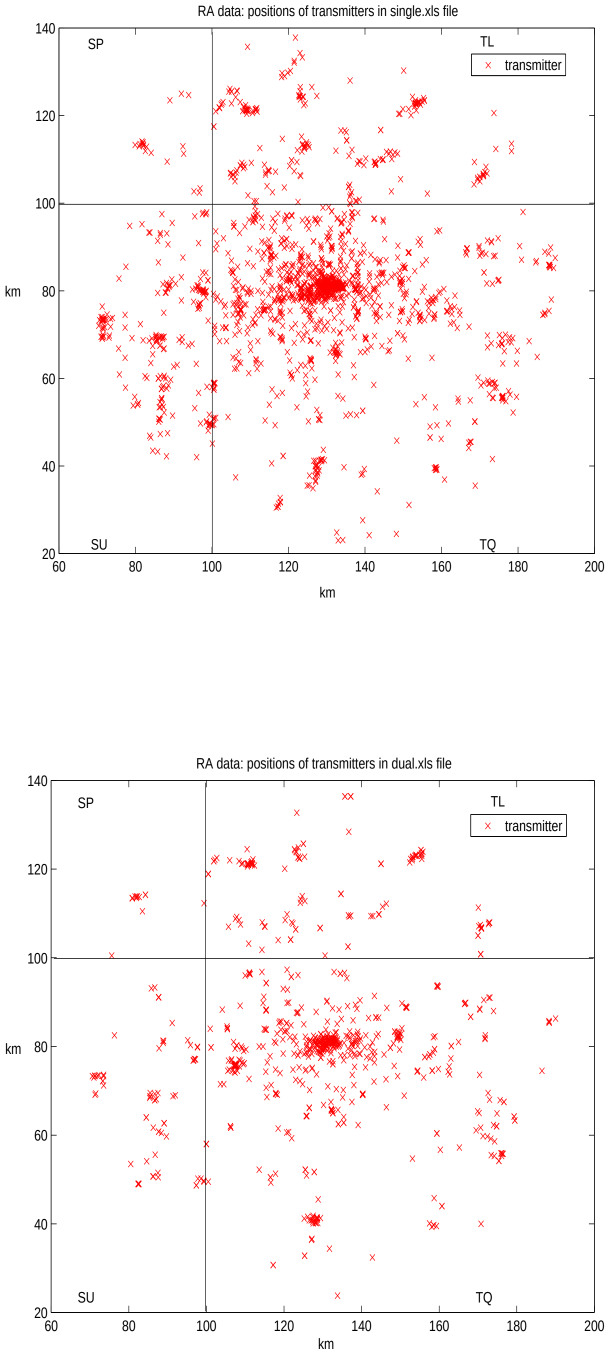

Figure 28: Distribution of single (top) and dual (bottom) licenses

2 Fixed spectrum modelling using the Agency's criteria

Although the Agency does not employ a detailed area coverage approach to assess their assignments, they do use a number of 'rules' in order to ensure that assignments are good enough to allow the services to operate. Satisfactory rules form the basis of simple constraints from which we can model and assess the Agency's private mobile radio assignments.

2.1 Constraints

We will consider three types of constraints: *re-use distance constraints, cosite constraints* and *channel loading constraints.*

- *Re-use distance constraints:* suppose that t_i and t_j are distinct transmitters from business classes b_i and b_j with co-channel re-use distances of d_i km and d_j km respectively. If the distance between t_i and t_j is found to be less than $\max\{d_i, d_j\}$ km then there must be at least 1 channel separation between t_i and t_j. This essentially means that here we are studying a pure graph colouring problem where transmitters correspond to vertices, edges correspond to separation between vertices and colours represent the frequencies.
- *Cosite constraints:* if cosited transmitters are assigned channels c_i and c_j respectively, then $|c_i - c_j| \geq x$. Currently the Agency take x to be 10. Transmitters are cosited if they are at the same national grid reference.
- *Channel loading constraints:* for each transmitter t_i the number of mobiles operating on the same channel as t_i within a distance of d km of t_i must be at most m. As a benchmark, the Agency takes d as 40 km and m as 200.

2.2 Frequency domains

The Agency originally allocated channel domains for each of the business classes. However, in updating the operational assignment plan, frequencies have been assigned outside these domains. This information is summarised in Figure 29, where the column 'ratio of freq to trans' gives an approximate measure of how difficult the task of channel assignment is, with ratio's closer to 1 indicating that the assignment procedure is likely to be easier.

It is interesting to note that in most of the current domains, even number channels are picked. This may have occurred in a bid to reduce adjacent channel interference within a given class, or historically, before channel spacing was reduced.

Business classes	Number of Trans	Original Domain Size	Current Domain Size	Ratio of Freq to Trans
O5	2220	52	83	0.037
D5	604	25	55	0.091
G3	204	13	53	0.260
H3	15	13	4	0.267
H4	6	12	5	0.833
M6	24	1	8	0.333
T1	49	12	15	0.306
U3	5	3	3	0.600
all dual	908	38	72	0.079
all single	2226	55	86	0.039
all licenses	3134	93	131	0.042

Figure 29: A summary of the channel domains currently used.

2.3 Minimum span

For each type of constraint, we have obtained bounds on the number of channels required. Lower bounds determine a theoretical minimum number of channels required for a given problem, while upper bounds correspond to the number of channels required for the best assignment obtained.

These are summarized in Figure 30. The bounds for the re-use constraints have been obtained using a block of contiguous channels. However, these results would be the same if non-contiguous channels were used as the re-use constraints only depend on whether channels are the same. The bounds on the cosite constraints have been obtained using the Agency's current domains. As with the re-use constraints, the channel loading constraints are independent of the spacing between channels and therefore contiguous blocks of channels have been used to generate these results.

Note that neither the cosite constraints or the channel loading constraints can be satisfied with the current number of channels. This means that we are dealing with a *fixed spectrum* channel assignment problem where our aim is to minimise some interference measure which is derived from making a constraint violation.

Bus. Class	Current Domain Size	Re-use ctr: Lower Bound	Re-use ctr: Upper Bound	Cosite ctr: Upper Bound	Loading ctr: Upper Bound
O5	83	389	389	83	650
D5	55	114	114	13	215
G3	53	18	18	12	56
H3	4	4	4	6	2
H4	5	1	1	6	1
M6	8	4	4	3	9
T1	15	3	3	8	12
U3	3	3	3	4	1
all dual	72	134	134	35	330
all single	86	390	390	45	650
all licenses	131	517	517	85	990

Figure 30: Bounds on the span required for each constraint.

2.4 Effect of re-use distance on span

The effect of redefining the re-use distance has a large impact on the span required. As an example, consider the dominant single and dual classes, O5 and D5 which both have a re-use distance of 6 km. Figure 31 shows the result of reducing this distance. When taking into account only re-use constraints, it is estimated that a re-use distance of approximately 1.2 km would need to be defined for the O5 and D5 classes to in order to achieve a zero violation assignment using the current channel domains.

	"O5" class		"D5" class	
Co-channel Protection	Lower Bound On Span	Upper Bound On Span	Lower Bound On Span	Upper Bound On Span
6 km	389	389	114	114
5 km	311	321	95	95
4 km	227	247	80	80
3 km	185	185	61	61
2 km	99	113	42	42
1 km	46	50	27	27

Figure 31: Effect of re-use distance on span for the O5 and D5 classes

3 Analysis of the Agency's current assignment

The Agency has supplied details of the current operational assignment. Using the constraints formulated in Section 2.1, we have carried out an assessment of this assignment.

3.1 Analysis of re-use constraint violations

The profile of the constraint violations is given in Figure 32. Here we have assessed the amount by which re-use constraints have been violated, classifying violations in 0.5 km intervals between 0 and 6 km. For example, a violation of 0 km occurs between a pair of transmitters when no re-use constraint is violated. Note that it is preferable to have small violations since such a violation corresponds to the re-use of a channel over a near satisfactory distance. Details of the average violations are given in Figure 33.

class	Instance of re-use violation (km)											
	0 to 0.5	0.5 to 1	1 to 1.5	1.5 to 2	2 to 2.5	2.5 to 3	3 to 3.5	3.5 to 4	4 to 4.5	4.5 to 5	5 to 5.5	5.5 to 6
O5	513	500	505	472	457	506	477	454	427	304	213	592
D5	60	24	56	33	54	46	46	56	37	33	22	108
G3	2	2	3	3	3	0	1	1	2	0	15	0
H3	0	0	0	0	0	0	0	0	0	0	11	0
M6	0	2	3	3	0	0	0	0	0	0	6	0
T1	0	0	0	0	1	0	0	0	0	0	0	0
dual	67	34	66	39	67	49	52	63	42	38	55	112
single	513	500	505	472	457	506	477	454	427	30	213	592
all	585	542	587	526	533	564	536	527	481	347	269	705

Figure 32: Re-use constraint violations: a profile of the current assignment

The profile of re-use constraint violations for all classes is plotted in Figure 36.

class	Number of Violations	Average Violation (km)	Sum of violated distance(km)
O5	5420	2.87	15570.91
D5	575	3.21	1848.04
G3	31	3.41	105.65
H3	11	5	55
H4	0	0	0
M6	14	2.89	40.44
T1	1	2	2
U3	0	0	0
dual	684	3.21	2195.2
single	5420	2.87	15570.91
all	6202	2.90	18011.33

Figure 33: Summary of re-use constraint violations

3.2 Analysis of cosite constraint violations

Should a pair of transmitters, t_i and t_j, have exactly the same national grid reference, then the cosite constraint imposes that the separation between the channels assigned to t_i and t_j should be at least 10. Figure 34 details all instances of co-located transmitters which have inadequate channel separations in the Agencies current assignment.

class	Sum of violns	cosite separation									
		0	1	2	3	4	5	6	7	8	9
O5	696	533	21	27	25	29	13	18	3	22	5
D5	122	101	0	5	0	7	4	5	0	0	0
G3	26	15	0	3	0	6	0	0	0	2	0
H3	11	11	0	0	0	0	0	0	0	0	0
H4	0	0	0	0	0	0	0	0	0	0	0
M6	9	6	0	3	0	0	0	0	0	0	0
T1	3	1	0	0	0	1	0	0	0	1	0
U3	3	3	0	0	0	0	0	0	0	0	0
dual	177	135	0	11	0	15	4	8	0	4	0
single	699	536	21	27	25	29	13	18	3	22	5
all	878	671	22	38	25	44	17	26	4	26	5

Figure 34: A profile of the violation of cosite constraints

The profile of cosite constraint violations for all classes is plotted in Figure 38.

3.3 Analysis of channel loading constraint violations

For each class, we have analysed the occurances of violated channel loading constraints. Taking each transmitter t_i on channel f_i, we have identified all transmitters within 40 km and calculated the total number of mobiles operating on channel f_i. Should this sum exceed 200 the channel is deemed *overloaded*. The *overload* for t_i is the excess number of mobiles operating from base stations within 40 km of t_i on channel f_i. The *total overload* is the sum of overloads for each transmitter. The distribution of overloads is presented in Figure 35. The large number of cosited transmitters allocated the same channel has the effect of violating a large number of channel loading constraints.

Class	Number Violating Trans	Total Overload	Instances of overload size						
			0 to 50	50 to 100	100 to 150	150 to 200	200 to 250	250 to 300	≥ 300
O5	1949	$2,983,371$	340	116	73	55	75	64	1226
D5	487	$220,776$	140	30	31	57	51	9	169
G3	57	7148	53	4	0	0	0	0	0
H3	12	682	12	0	0	0	0	0	0
H4	0	0	0	0	0	0	0	0	0
M6	9	1066	9	0	0	0	0	0	0
T1	5	275	5	0	0	0	0	0	0
dual	691	$281,986$	231	47	53	62	71	31	196
single	1949	$2,983,371$	340	116	73	55	75	64	1226
all	2699	$3,350,198$	554	169	121	126	158	98	1473

Figure 35: A profile of the violation of channel loading constraints

The profile of cosite constraint violations for all classes is plotted in Figure 40.

4 New Assignments

In this section we outline the different approaches which have been used to obtain new assignments.

4.1 Using FASoft

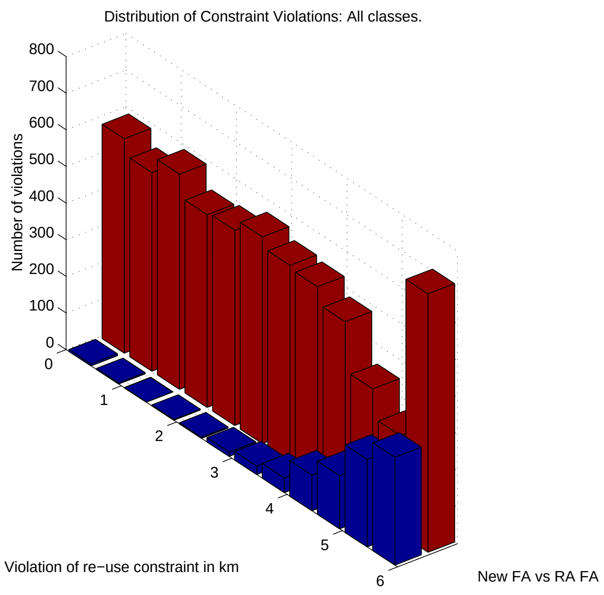

Figure 36: A profile of violations for all classes: FASoft assignment verses RA current assignment

The software package FASoft [5] is capable of solving channel assignment problems which are characterised by *binary constraints*, which impose necessary channel separation between pairs of transmitters. The cosite constraints and re-use distance constraints can both be formulated using binary constraints. However the channel loading constraints cannot be formulated as binary constraints, and are therefore called *non-binary constraints*.

The FASoft package has been used to solve the re-use distance constraints. A profile of the violations the FASoft assignment makes is compared with those of the current Agency assignment in Figure 36.

Although the number of violations in the FASoft assignment are significantly fewer, the profile of violations is undesirable, with large violations being favoured to small ones. This characteristic is due the *cost function* which the FASoft package uses. The cost function treats all violations equally, however large or small.

In order to re-profile the violation of constraints, it is necessary associate a larger cost the larger the constraint violation. This cannot easily be incorporated into the existing FASoft code.

4.2 Generating constraints

A common approach when solving channel assignment problems is to initially begin by identifying and recording all possible constraints. The search can then proceed by checking whether or not these constraints have been violated for any particular assignment. This method is used in the FASoft package [5] for example. However, as the channel loading constraints are non-binary constraints, the number of ways of satisfying (or violating) them is large compared to binary constraints. Therefore the computational cost in generating and storing them is considerable.

A program has been written to generate and represent channel loading constraints in a useful format. As no suitable algorithm could be found in the literature, a method was devised based on an algorithm by Carraghan and Pardalos [2] to find maximal cliques. An example of the type of constraint generated is the following:

$$0\ 1\ 2\ 10\ 12 < 7\ 9\ 11\ 24\ 36\ 45.$$

Here the transmitters 0,1,2,10,12 form a *maximal co-channel set.* This set of transmitters can be on the same channel f without any transmitter incurring channel overloading if and only if none of the transmitters 7,9,11,24,36,45 are on channel f.

A large number such constraints exist, for example the T1 taxi class has approximately 250,000 such constraints and yet contains only 49 transmitters. This makes the approach of pre-generating all possible constraints for large problems such as the UHF 2 band problem infeasible.

4.3 A new approach

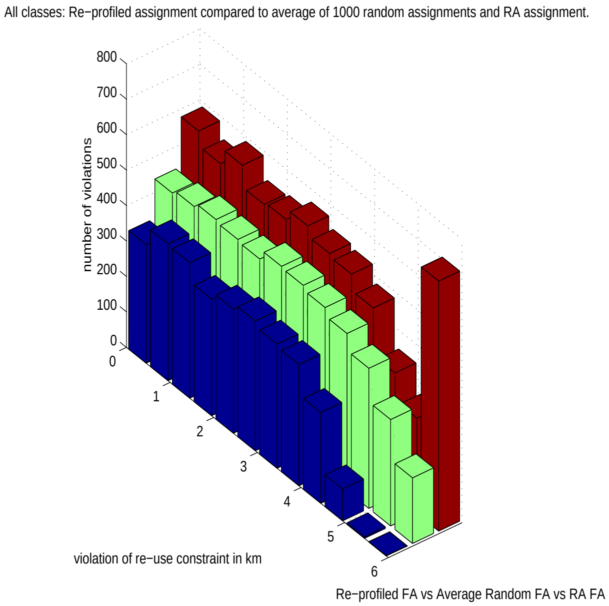

Figure 37: A profile of re-use distance violations for all classes: re-profiled assignment compared with average of 1000 random assignments compared with RA assignment

A program has been written based on the *tabu search algorithm* to simultaneously solve re-use distance constraints, co-site constraints and channel loading constraints. To tackle the problem of pre-generating all channel loading constraints, we have adopted a *constraint free* approach. This means that we search for solutions without first identifying all the constraints, and instead, we directly test possible solutions for satisfaction of the constraint criteria as required.

The program allows the user to prioritize particular constraints according to their own preference. In addition, for each type of constraint, large violations are penalized more than small violations. The cost of a particular assignment is calculated using the cost of the previous assignment considered, by noting which transmitters have changed frequencies. This is known as an *updating cost function.*

The program has been used to determine the upper bounds on the spectrum required for each of the three types of constraints. It addition, we have run the program to obtain an assignment with the violation characteristics as displayed in Figures 37, 38 and 40. In each of these figures we compare our assignment with both the average of 1000 random assignments and the Agency's current assignment. In obtaining our assignment, we have employed the 131 channels currently used by the Agency. The comparison is as follows:

- The total sum of re-use distance violations is 6667.96 km for the re-profiled assignment compared to 13928 km for the average random assignment and 15570.91 km for the Agency's current assignment.
- The total number of cosite violations is 99 for the re-profiled assignment compared to 220 for the average random assignment and 878 for the current RA assignment.
- The total overload is 925,230 for the re-profiled assignment compared to 1,044,340 for the average random assignment and 3,350,198 for the current RA assignment.

The relatively poor performance of the Agency's assignment is likely to be a reflection of the fact that the Agency's assignment procedure is effectively sequential and in addition, there may be other criteria which the Agency implicitly use when making assignments.

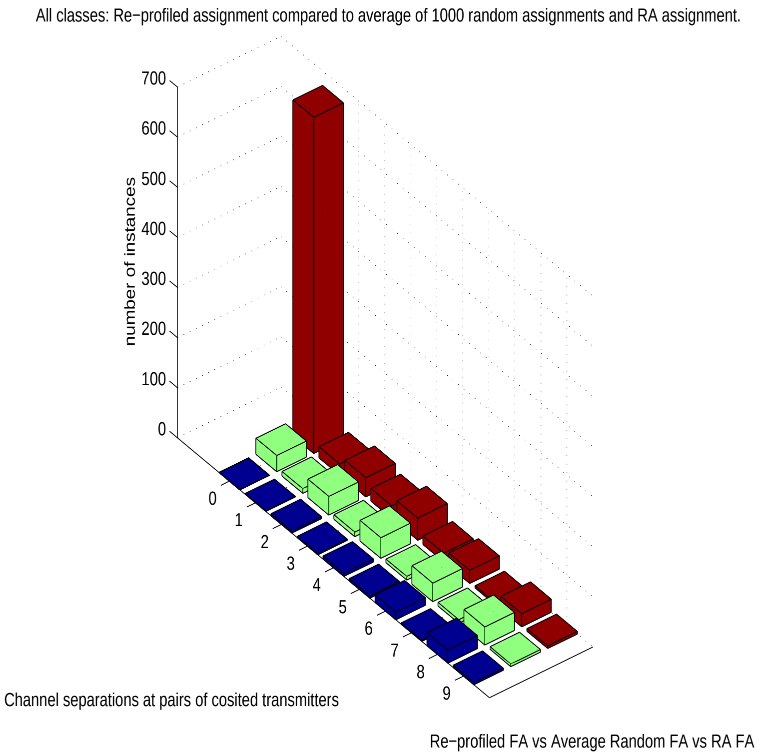

Figure 38: A profile of cosite violations for all classes: re-profiled assignment compared with average of 1000 random assignments compared with RA assignment

Note that the RA assignment has a disproportionate level of cosited transmitters on the same channel (i.e. 0 channels separation) in Figure 38. Also the frequency of violations alternates across channel separations. This is because the Agency have alloted more even numbered channels than odd. If we disregard the 0 channel separation cosited transmitters, as in Figure 39, we can see a better comparison for the other channel separations.

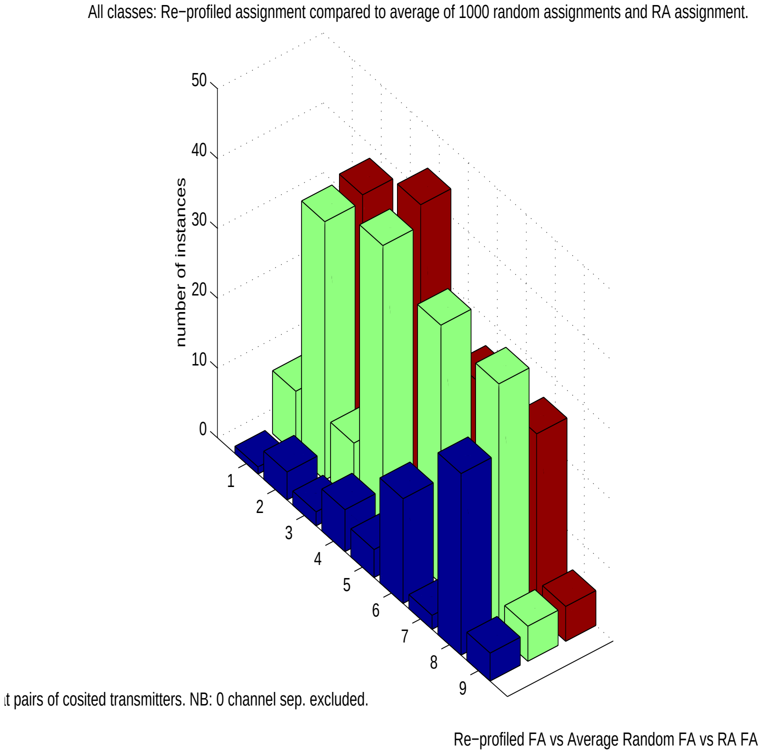

Figure 39: A profile of cosite violations for all classes: re-profiled assignment compared with average of 1000 random assignments compared with RA assignment

It appears that while channel loading constraints and cosite constraints work well together, channel loading constraints and re-use distance constraints are slightly antagonistic. Channel loading constraints and re-use distance constraints are related as channel loading constraints essentially “cap” the number of times a channel can be reused within 40km of a transmitter.

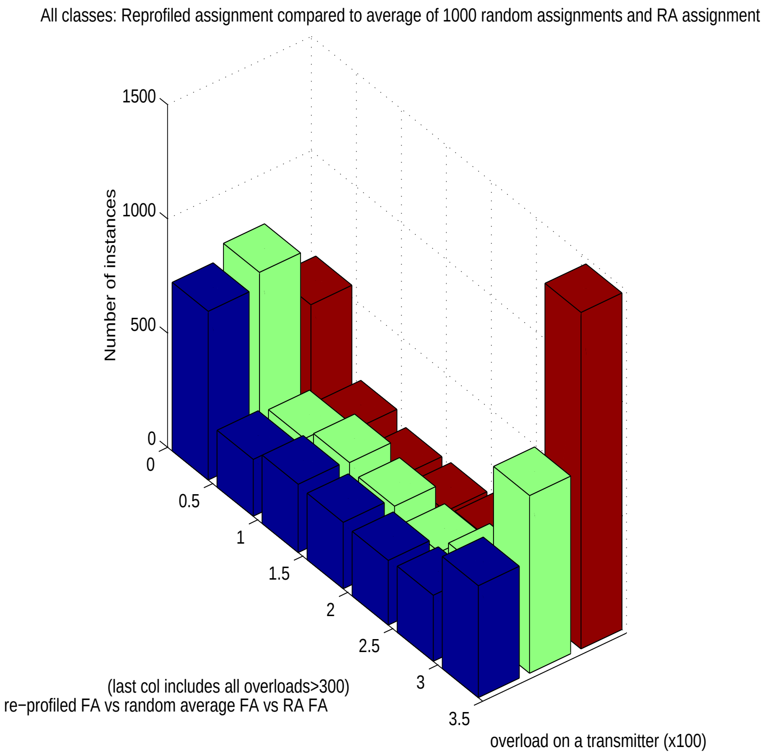

Figure 40: A profile of cosite violations for all classes: re-profiled assignment compared with average of 1000 random assignments compared with RA assignment

4.4 Number of channels required

Taking each of the constraint types in isolation, we have generated assignments which approximately match the Agency's current violations. This information is summarized in Figure 41. Note that for the both the channel loading and re-use distance constraints, the results on span are independent of whether the channels used are contiguous. For the cosite constraint, we have used both contiguous and non-contiguous channels.

Bus. class	Channels for reuse ctr	Channels for loading ctr	Channels for cosite ctr (contig. chls)	Channels for cosite ctr (non-contig. chls)
O5	≤ 44	≤ 35	≤ 40	≤ 15
D5	≤ 26	≤ 24	≤ 15	≤ 5
all dual	≤ 35	≤ 40	≤ 20	≤ 10
all single	≤ 45	≤ 35	≤ 35	≤ 15
all licenses	≤ 85	≤ 55	≤ 55	≤ 30

Figure 41: A summary of the number of channels required to match violations made in the Agency's current assignment.

In practice we need to consider all three types of constraint simultaneously. An unweighted sum of all three types of violation has been used to obtain an assignment comparable to the current operational assignment. This assignment was obtained in 90 channels which compares favourably with the 131 currently used. This result clearly indicates the benefits of the methods described in this report.

5 Conclusions

Analysis of the current assignment has demonstrated that heuristic techniques can be employed to significantly improve the current operational assignment. The results suggest that there is significant scope to improve the operational assignment when the band is realigned. We conclude that it is possible to reduce the number of channels needed for PMR services without compromising the quality of service provided. In the following Appendix (Section 6) we describe an alternative approach to modelling PMR services, by using an area coverage model.

6 Appendix: An area coverage model for the UHF 2 band

At an early stage in the research, an *area coverage* approach was used to model private mobile radio (PMR) services in the UHF 2 band. This model simulates the propagation of the wanted signal taking into account the effect of interfering signals, for each reception point set down in the plane. The results that are obtained from this model imply that satisfactory assignments require a large number of channels. However, this general form of model could be improved upon if equitable sharing of channels in time could be established between interfering licenses. This is done currently in the channel loading constraint. Some of the assumptions in this model, such as the value of θ, require modification.

6.1 Choice of reception points

Reception points have been taken as the points on a regular mesh with horizontal and vertical separations of 1km which both:

- fall within a radius d km of transmitter T where d is the co-channel re-use distance associated with the license using the transmitter T
- do not correspond to transmitter locations.

The single licenses have 11,127 reception points and dual licenses have 8,130 reception points. It is assumed that the height of each reception point is 1.5 m.

6.2 Propagation Model

To simulate the propagation of radio signals, a model due to Hata [4] is used. This statistically based algorithm is used in calculating the signal strength of the wanted transmitter and the interference contribution of an unwanted transmitter. The path loss at a reception r_i from transmitter t_j, denoted L_{ij}, is given as:

$$69.55 + 26.16 \log(f) - 13.82 \log(h_b) - A + 44.9 - 6.55 \log(h_b) \log(d) \text{ dB}$$

where

$$A = 1.1 \log(f) - 0.7h_m - 1.56 \log(f) - 0.8$$

and:

- f, the channel, is taken mid-range: ie 460 MHz
- h_b, the height above sea level of t_j, is taken as sum of antenna height and terrain height. When this sum is zero the average of the given antenna heights is used.
- d, the distance from r_i to t_j is in the range 1 to 20 km.
- h_m height of mobile (r_i) is taken to be 1.5 m.

Suppose that reception point r_i is tuned to transmitter t_j where p_j is the power of the signal from transmitter t_j. Then the signal strength S, at r_i is assumed to be:

$$S = \frac{p_j}{10^{\frac{L_{ij}}{10}}}.$$

The interference contribution at r_i, I_{ik}, from an unwanted transmitter t_k is given by:

$$I_{ik} = \frac{p_k}{10^{\frac{L_{ik}}{10}}}\theta$$

where $\theta = 1$ if t_j and t_k are co-channel otherwise

$$\theta = 10^{\frac{-15(1+\log_2 \delta f)}{10}}$$

where δf is the channel separation between the wanted transmitter t_j and the unwanted transmitter t_k. This model is a simple adaptation of that used by Leese and Gower [3]. We note that the value of θ given by the Leese and Gower model is not typical when considering PMR equipment.

6.3 Constraints

Many different but equally reasonable assumptions can be made when constraints are generated for the single and dual licenses. These assumptions are responsible for variations in the final span reported. Three important decisions which must be made concern:

- **co-site constraints**
  It is assumed that co-sited transmitters require at least 10 channels separation. However, as service areas frequently overlap, it is unclear as to how close transmitters should be before we consider them as being co-sited. The distance chosen has a significant impact on the span. If transmitters within 0 km of each other (i.e. same location) are to be taken as co-sited then co-site constraints give rise to a lower bound of 160 on the span. If this distance increases to 1 km then the lower bound from co-site constraints rises to 460. A distance of 1.5 km gives a lower bound of 780 while a 2 km co-sited distance gives a lower bound of 1050.

- **adjacent channel constraints**
  It is frequently the case that for a reception point to have adequate signal-to-interference ratio (ie a satisfactory level of service), a separation of more than one channel is required between certain transmitters. However, the problem as stated in [1] has no adjacent channel constraints.

- **propagation model**
  The Hata model requires parameter d, the distance between the transmitter and receiver, to be in the range 1 to 20 km. Clearly this is not going to be the case given that we are working within a 60 km radius of central London. We may safely neglect the case when d is greater than 20 km as the signal strength would have fallen off significantly, due to distance. However, cases where d is less than 1 km are important. We have a number of possibilities:

  1. if $d < 1$, omit using this reception point/transmitter combination in constraint generation,
  2. put d into Hata path loss formulae even if it is less than 1,
  3. if $d < 1$ then assume $d = 1$ in the Hata path loss formula.

6.4 Results

Software has been written to generate binary constraints adopting the single interferer assumption at the required signal-to-interferer ratio (SIR). Assignments have been found using FASoft.

In Figure 42 a summary of the results are displayed. 'Co-site Distance' is the distance within which transmitters are assumed to be co-sited. 'Accurate Adjacent Channel Sep.' documents whether separations of greater than 1 are to be used when generating constraints, and 'Prop. Model Assumption' records how d is treated in the Hata model (See section 6.3: Propagation model). The dual problem requires an assignment of a channel pair (transmit and receive). Constraints for this problem have been generated assuming that only a transmit assignment is required and therefore the associated assignment needs to be copied and the associated span doubled. In a bid to save spectrum, the single and dual licenses have been simultaneously considered in Case 8) of Figure 42.

Note that the upper and lower bounds have been generated using a simple clique algorithm. It is likely that these could be improved upon and that the upper bounds can be reduced with long runs using FASoft.

Case	Required SIR (dB)	Co-site Distance (km)	Accurate Adjacent Channel Sep.	Prop. Model Assumption	Span: Lower Bound	Span: Upper Bound
1) Single	12	0	No	1	160	743
2) Single	12	1	Yes	3	490	1280
3) Single	0	1	Yes	1	572	646
4) Single	0	1	Yes	3	514	818
5) Dual	12	0	No	1	110	271
6) Dual	12	1	Yes	3	410	460
7) Dual	0	1	Yes	3	410	410
8) Combined	12	1	Yes	3	660	1690

Figure 42: Spans for Single and Dual Problems

6.5 Minimum span

Cases 1) and 5) from Figure 42 combine to give a span of $\leq 743 + 2 \times 271 = 1285$ for the original problem as stated by the Radiocommunications Agency [1]. However, since there are no adjacent channel constraints many reception points are left with inadequate separations between wanted and unwanted signals. This is investigated more closely in Section 6.6.

Cases 2) and 6) allow accurate adjacent channel separation to occur. This gives a span of $\leq 1280+2\times 460 = 2200$ but it makes sure that every reception point has adequate separations between wanted and unwanted signals.

Case 8) demonstrates that there is a gain of 50 channels from considering the single and dual problems simultaneously. Solving the problems separately gives a span of 2200 (cases 2) and 6)) while cases 2) and 8) combine to give a span of $1690 + 460 = 2150$.

6.6 Area Coverage

The assignments made in Section 6.4 have been assessed for adequate area coverage. For a chosen test SIR α, the percentage of reception test points meeting α has been identified, using both a single interferer assumption and a multiple interferer assumption. Coverage calculations have been made with the propagation model Hata, using assumption 3 with respect to distances (See section 6.3: Propagation model). A summary of the main results is given in Figure 43.

Constraints (From Fig. 1)	Constraints Generated at: (dB)	Span	Test SIR level (dB)	Single Intf. Coverage	Mult. Intf. Coverage	Notes
Case 1). (Single)	12	≤ 743	12 15	74.4% 55.1%	67.3% 49.8%	RA Model
Case 2). (Single)	12	≤ 1280	12 15	100% 97.8%	97.9% 90.6%	
Case 5). (Dual)	12	≤ 271	12 15	73.1% 54.9%	67.8% 50.8%	RA Model
Case 6). (Dual)	12	≤ 460	12 15	100% 96.9%	97.2% 89.2%	

Figure 43: Coverage for Single and Dual Licenses

The relatively small span obtained for cases 1) and 5) is reflected in lower levels of coverage. Here many reception points are receiving inadequate levels of service due to the lack of accurate adjacent channel constraints.

The multiple interference coverage results are surprisingly good, despite the constraints being generated using only the single interferer assumption. This can be explained by the high span relative to the number of transmitters. The dominating interferers are usually those which are co-channel with respect to the serving transmitter. However, in the single and dual problems channel reuse is relatively small. For example, in Case 2), where the coverage (multiple interferer) is 97.9% at 12 dB, each channel is used an average of just 1.7 times. This is contrasted against Case 1) where each channel is used approximately 3.0 times and the coverage (multiple interference) at 12 dB falls to 67.3%.

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