Radiocommunications Agency Agreement RCCM070

Final Report

Evaluation of Models for Area Coverage

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Executive Summary

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Algorithms for frequency assignment have traditionally started from a set of constraints which define the necessary minimum separation between the channels assigned to a given pair of transmitters. Such constraints, referred to as *binary constraints*, are based on a model which takes no account of multiple interference. Such *binary models* may not provide 100% coverage (in terms of adequate signal-to-interference ratio) if multiple interference is considered.

It has been demonstrated in previous reports that given a predefined set of binary constraints it is possible to both find good assignments and to determine the necessary spectral requirement (or *span*) with surprising accuracy. Use of a non-binary model may provide better coverage, but new algorithms to find assignments and new methods to determine the span (for 100% coverage) become necessary. In this document some algorithms and methods are described and compared.

The models are fully described in Section 2 and the results for test problems are given in Section 3. Section 3 also includes the results of a preliminary investigation into the possibility of using these methods to determine the span of networks on the basis of some parameters.

A heuristic based directly on the non-binary model does sometimes produce better assignments than the traditional binary approach, but for problems of a realistic size the run time becomes prohibitive. A modification of the commonly used sequential approach, which builds experience on the hardest transmitters to assign and reorders transmitters accordingly, is shown to perform well, although it is again too slow for large problems in its current form. A method using binary constraints in which the constraints are selectively strengthened is close to equalling the best performance of the non-binary heuristic for small problems, and exceeds its performance for large problems. Its run time is practical even for large problems.

Methods of determining lower bounds for the span developed for the binary model are still applicable for the non-binary model, and can be used to assess how close a particular span achieved is to best possible.

In the Conclusions several potential improvements are identified. The use of non-binary

constraints globally may prove too expensive, but they may prove useful selectively to improve the binary constraint strengthening approach. There is also considerable potential to use them to improve the lower bounding method. Additionally, it may be possible to improve the speed of the sequential method substantially, by using binary constraints to eliminate some possible assignments to a transmitter without performing signal-to-interference calculations. More realistic results could be obtained by replacing the simple inverse fourth power law by a more accurate propagation loss algorithm. When these improvements have been carried out the results can be used to refine the techniques for determining the span of networks on the basis of parameters. The parameters themselves can also be more closely related to the real world problem.

It seems reasonable to expect that when these improvements are completed it will be possible to determine the necessary spectral requirement for 100% coverage quite accurately. It should also be possible to determine an assignment of minimal or near minimal span for any given level of coverage.

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1 Introduction

This work is a collaboration between the Universities of Cardiff and Glamorgan and is funded by the Radiocommunications Agency under agreement RCCM070.

Algorithms for frequency assignment have traditionally started from a set of constraints which define the necessary minimum separation between the channels assigned to a given pair of transmitters. Such constraints, referred to as *binary constraints*, are based on a model which takes no account of multiple interference. Such *binary models* may not provide 100% coverage (in terms of adequate signal-to-interference ratio) if multiple interference is considered.

It has been demonstrated in previous reports that given a predefined set of binary constraints it is possible to both find good assignments and to determine the necessary spectral requirement (or *span*) with surprising accuracy. Use of a non-binary model may provide better coverage, but new algorithms to find assignments and new methods to determine the span (for 100% coverage) become necessary. In this report some algorithms and methods are described and compared.

The propagation model used will be described first and then a heuristic algorithm based directly on this model will be described. An alternative non-binary version of a sequential (greedy) algorithm is also described. This is based on an idea first used in [7] of allowing the algorithm to iteratively build up experience of the difficult transmitters to assign and using the results to determine a new transmitter ordering. The traditional binary method with global increases in the signal-to-interference ratio used is described, as well as a version in which constraints are strengthened selectively.

Additionally, it is pointed out that a lower bound for the span for 100% coverage can be determined from binary constraints generated at the appropriate signal-to-interference ratio.

Results will be presented for these methods for several networks. Attention will be given to the practicability of the algorithms for problems of a moderately large size.

An attempt will then be made to determine a formula for the span of a network in terms of some parameters which describe it. This work is based on the technique of *multiple regression*.

2 Methods

2.1 Introduction

In this section, a number of frequency assignment methods are described. Two of these use binary constraints and two use a multiple interferer approach. The same simple propagation model is used, both by the binary constraint methods to calculate the constraints and by the multiple interferer methods to explicitly calculate signal and interference strengths. This model is described below.

2.2 Propagation Model

The simple model employed calculates both the signal and interference strengths at any point. It takes no account of terrain or of any factors other than a path loss dependent on distance and frequency separation. The signal and interference strengths at a point are given as follows. If reception point r_i, where $i = 1, ..., N_r$, is tuned to transmitter T_k, where $k = 1, ..., N_t$, then the signal strength S_i, at r_i is assumed to be given by:

$$S_i = \frac{P_k}{d_{ik}^{\gamma}}$$

where P_k is the power of the transmitted signal, and d_{ik} is the distance between transmitter T_k and the receiver r_i. For simplicity, P_k is assumed to be the same for all transmitters, and $\gamma = 4$ is used in this model. The total interference I_i at reception point r_i is given by:

$$I_i = \sum_{\substack{j=1 \\ j \neq k}}^{N_t} \frac{P_j}{d_{ij}^{\gamma}} \theta$$

where N_t is the number of transmitters and, for each j, θ is assumed to be taken as

$$\left.\begin{array}{l} \theta = 10^{\frac{-\alpha(1+log_2 \delta f)}{10}} \\ \theta = 1 \end{array}\right\} \text{ if } \delta f \left\{\begin{array}{ll} \neq 0 & \text{(adjacent channel)} \\ = 0 & \text{(co-channel)} \end{array}\right.$$

where δf is the channel separation between desired transmitter T_k and interfering transmitter T_j. α is an attenuation factor for adjacent channel interference and is measured in dB/octave. This simple model was first proposed by Leese and Gower [4] for use in this type of evaluation.

2.3 Non-Binary Methods

2.3.1 Heuristic Approach

The Non-Binary heuristic approach involves iteratively selecting and testing frequency assignments for the required signal to interference ratio (SIR) at a set of reception points in the area covered by the network. During each iteration the desired signal strength and interference from some or all of the other transmitters is calculated at each reception point. Coverage at a reception point requires that each of the one or more wanted signals from the one or more designated transmitters meets the required SIR, denoted by σ, when the total interference at that point is taken into account. The frequency assignment is then deemed a successful solution if coverage over all (or some defined percentage) of the reception points is achieved.

The Non-Binary method begins by finding an initial assignment that is a solution with a particular span, say q, which is the difference between the number of the largest channel used and the number of the smallest channel used. All transmitters which are assigned the highest frequency channel are reassigned to random frequencies in the range $[1, q]$. A simulated annealing heuristic is used to find an assignment with span at most $(q-1)$. The procedure of iteratively reducing the span continues until an assignment cannot be found at a given span. The progress of the search is guided by the simulated annealing metaheuristic. This is based on the physical process of annealing in which metals are heated and then allowed to cool until they reach a minimum energy state. The equivalent energy or cost function E used in this non-binary method is given by:

$$E = \sum_{i=1}^{N_r} \left(\sigma - \frac{S_i}{I_i} \right)^a \text{ including only terms with } \frac{S_i}{I_i} < \sigma$$

The value of a is taken to be 2 for the results presented here.

When the simulated annealing meta-heuristic is used to minimise E, an improvement in E is always accepted but instances when E doesn't improve can also be accepted with a certain probability given by $e^{-\frac{\Delta E}{t}}$, where ΔE is the change in the cost function of the current assignment from the previous assignment, and t is an appropriate temperature which diminishes with time. So if *random* is a random number within the range $[0, 1]$ then an assignment that doesn't improve is accepted if $random < e^{-\frac{\Delta E}{t}}$. This allows the method the possibility of escaping from local minima.

In each iteration a new assignment is found within a neighbourhood of the current assignment, where the neighbourhood is the set of all assignments that can be obtained from the current assignment by changing the frequency allocated to one transmitter only. A proportion of the new assignments are formed by changing the frequency allocated to a transmitter that contributed to the total interference at one or more violating reception points. The remainder are chosen randomly. The proportion used was often 50%, but other values were used for the results presented here.

When E is zero then an assignment has been found at the current span. Then the transmitters assigned the highest channel are randomly reassigned lower channels and the search is repeated. Pseudo code is given in Figure 1.

```
Input transmitter coordinates
Input numrec receiver coordinates
Produce an initial assignment of span q from the ordering of
the transmitters using a sequential algorithm.
WHILE (termination criteria not met) DO
      Reassign transmitters with frequency q+1 to random frequency within [1,q]
      Calculate coverage and total cost (E) over all violating reception points

      WHILE (coverage < 100%) DO
            Select transmitter (or violating transmitter) at random
              and change its frequency assignment within [1,q]
            Calculate new coverage and change in cost, ΔE

            IF (coverage < 100%) THEN
               Calculate new simulated annealing temperature t

               IF (ΔE < 0) OR (random < e^(-ΔE/t)) THEN
                  Accept new assignment
               ELSE
                  Return to previous assignment
               END IF

            END IF
      END WHILE

END WHILE
```

Figure 1: *Non-binary heuristic approach*

Variations of the algorithm with a criterion "*coverage* $< p\%$" (for some $p < 100$) are also used.

This method is computationally intensive and the run time may be prohibitive. Note that to minimise the run time it is necessary to arrange the calculation of the cost function so that only those parts that involve frequency are recalculated at each iteration. Even when this is done, the time to obtain the lowest span for large networks may be several days.

2.3.2 Sequential Approach

In [7], Thavarajah and Lam introduced a new approach to the design of sequential (or greedy) algorithms. Following an initial ordering of transmitters, the algorithm iteratively determines the transmitters which are hard to assign by virtue of the fact that they are assigned high channel numbers. This leads to an ordering which is determined by experience instead of by some initial assumption as to what constitutes a good ordering. This approach has some potential advantages for the multiple interferer model described here. A variation of the method introduced in [7] is described below.

The transmitters are initially ordered in some way, *e.g.* by largest degree. The first transmitter is assigned channel 1. The second is then also assigned channel 1 and the signal to interference ratio at each receiver assigned to either transmitter is calculated. If the SIR does not exceed the required level for any receiver, then the channel assigned to the second transmitter is increased by 1, the interference is calculated again and so on until there are no reception points at which the SIR is inadequate. Transmitter 3 is added and allocated the channel 1 and the interference over all receivers allocated to the first three transmitters is calculated. The channel allocated to transmitter 3 is again increased until again there are no violating reception points. This continues with each transmitter being added in turn until a complete assignment is achieved.

This first assignment is often poor in terms of span, however the assignments obtained can be used to produce a new ordering that may lead to a better assignment. In an iterative loop, the η transmitters allocated the highest channel numbers in the last assignment should be amongst the most difficult to assign and are therefore placed at the start of the next ordering. The remaining transmitters follow with their order unchanged. The number η of transmitters to be reordered in each iteration is chosen at run time and is dependent on the total number of transmitters; a value of 5% of the number of transmitters is typical. It may be beneficial to vary the number reordered with time, possibly with a reduction guided by a metaheuristic such as simulated annealing. This will be attempted in the future. Pseudo code for the algorithm implemented is given Figure 2.

```
Input N_t transmitter coordinates, N_r receiver coordinates,
required SIR, the number N_change of transmitters to be reordered at
each iteration and an inital ordering

FOR(N iterations) DO
   Input current ordering
   Assign channel 1 to the first transmitter
   Set total interference to zero for all receivers
   FOR (i = 2 to N_t) DO
       Set initial channel, C_i = 0
       Calculate signal strengths at all reception points tuned to i
       SIR_min = 0

       WHILE(SIR_min < SIR) DO
            C_i = Ci_+1
            j = 1
            SIR_min = SIR
            WHILE(SIR_min ≥ SIR) AND (j ≤ i) DO
                 FOR (k = 1 to number of receivers tuned to j) DO
                     Calculate increase in interference, I_jk, at jk
                     Calculate signal to interference = S_jk/(I_jk tot + I_jk)
                     IF (S_jk/(I_jk + I_jk tot) < SIRmin) THEN
                          SIR_min = S_jk/(I_jk + I_jk tot)
                     END IF
                 END FOR
                 j = j + 1
            END WHILE

       END WHILE
       Add transmitter i to the appropriate place in a separate list of the transmitters
       positioned in descending order by channel

   END FOR
   Produce a new ordering by moving the N_change highest
   assigned transmitters to the top of the next order, keeping
   the N_t − N_change remaining transmitters in the same order.
END FOR
```

Figure 2: *Non-binary Greedy method*

2.4 Binary Method

2.4.1 Introduction

The frequency assignment system FASoft [5], in common with many frequency assignment algorithms, produces frequency assignments from a constraint (or channel separation) matrix. This matrix is formed from binary constraints that are produced for pairs of transmitters. The individual constraints are calculated on the basis of a particular required SIR. The constraints ensure that neither transmitter in the pair acts as a violating interferer at any point at which the other transmitter provides the desired signal. The constraints are of the form $|f(T_i) - f(T_j)| \geq c_{ij}$ where $f(T_i)$ and $f(T_j)$ are the frequencies assigned to transmitters i and j and c_{ij} is the value that the channel separation between transmitters i and j must exceed.

The system again employs heuristic methods (e.g. simulating annealing or tabu search) to iteratively search for an assignment that does not violate the constraints. When a solution is found at a particular span, the largest channel is removed and the search begins again. FASoft can, if required, utilise techniques taken from graph theory to exploit the fact that often transmitters that are hard to assign form fully connected subgraphs (cliques) and it is often advantageous to assign the transmitters within these cliques before attempting to assign the whole problem.

As each constraint involves only two transmitters at a time, some pertinent interference information is inevitably omitted from the calculation of the required channel separations. As a consequence of this, assignments produced from constraints calculated at a particular SIR will rarely produce 100% coverage. FASoft and related systems are able to generate lower bounds for the span of an assignment given all of the binary constraints the assignment has to satisfy. An important observation is that this lower bounding technique can be applied to the non-binary method. Suppose that for a particular arrangement the cost function E is zero (so all necessary non-binary constraints involving more than one interferer are satisfied for the defined SIR at the defined reception points). Then it follows that the binary constraints generated at the same SIR using the same reception points must also be satisfied (the reverse is not true). Thus the span for the non-binary model with 100% coverage satisfies the same lower bound. Perhaps surprisingly, this lower bound is sometimes found to be fairly tight. This simple but previously unnoticed observation brings the evaluation provided by the lower bounding methods to the non-binary case.

2.4.2 Constraint generation

Binary constraints are produced between pairs of transmitters at a particular required SIR. They are required to ensure that the interference contribution from either transmitter does not provide violating interference at any reception point tuned to the other transmitter. Put another way, the signal to interference ratio at any reception point tuned to either transmitter must exceed the required level when only the interference from the other transmitter is considered. The method employed is described below.

Constraints for all combinations of pair of transmitters are calculated in turn. Despite there potentially being a large number of receivers tuned to one or other of the transmitters, there will be only one that is used to calculate the constraint. This must be the worst case receiver *i.e.* the receiver that has the worst potential signal to interference ratio as calculated by the propagation model used. With the propagation model employed in the current work, the signal and interference strengths for given frequencies fall off with distance only. The worst case receiver then, is that which has the lowest ratio of distance to tuned transmitter to distance to interfering transmitter.

Once this worst case transmitter has been identified its signal to interference ratio can be calculated for any frequency separation between the interfering and tuned transmitter. So, starting with a frequency separation of zero and then, if necessary, repeatedly increasing the separation by 1 until the SIR is greater than the required level, the minimum separation for non-violating interference can be found. If, for example, the minimum separation between transmitters 2 and 5 was found to be 2, then the binary constraint between them would be written (in the commonly used CELAR format) as follows: `2 5 > 1` This can be interpreted as stating that the necessary channel separation between transmitters 2 and 5 must be greater than 1. The coding structure is shown in the pseudo code Figure 3.

Input N_t transmitter coordinates
Input N_r receiver coordinates
FOR $i = 1$ to N_r DO

    For each receiver i identify its tuned transmitter T_t.

    For each interfering transmitter T_n, calculate the ratio, R_{tn}, of the distance from i of T_n to the distance from i of T_t.
    IF ($R_{tn} < R_{tn}^{min}$) THEN
        $R_{tn}^{min} = R_{tn}$ where R_{tn}^{min} is the minimum value of R_{tn} over all receivers tuned to transmitter T_t
    END IF

END FOR

FOR $j = 1$ to N_t DO

    FOR $k = j + 1$ to N_t DO

        $R_{jk}^{min} = Min(R_{jk}^{min}, R_{kj}^{min})$
        $R_{jk} = R_{jk}^{min}$ raised to the power 4
        $DF = 0$
        $R_{jk}^{new} = R_{jk}$

        WHILE($R_{jk}^{new} < SIR$) DO
            $DF = DF + 1$
            $R_{jk}^{new} = R_{jk} * 10^{\frac{\alpha(1+log_2 DF)}{10}}$
        END WHILE

        IF ($DF > 0$) THEN
            Output $(DF - 1)$ as the constraint between transmitters j and k
        END IF

    END FOR

END FOR

Figure 3: *Generation of binary constraints*

2.4.3 Automated Constraint Strengthening

The method described in Figure 3, produces assignments from binary constraints produced on the basis of a single required signal to interference level. It is possible that better assignments could be produced from constraints calculated at a number of signal to interference levels. This means that for an assignment to be successful at a particular SIR, the binary constraints that produce it must be generated at a variety of SIRs instead of the single one used in Figure 3. This can be done by selectively strengthening some constraints *i.e.* adding 1 or more to the channel separation they dictate. An automated way of producing assignments and determining which constraints should be strengthened is described below.

If an assignment that produces a certain level of coverage over a network is required then, as above, the process starts with an assignment produced by the constraint solver using binary constraints calculated for a particular SIR. When assessed over the test grid (see Section 2.5) this assignment may not produce the required coverage. Analysis of the coverage produced by this assignment identifies the test points at which coverage is not achieved and also the primary interferer which is the greatest contributor to the violating interference. Hence for each violating test point, its tuned transmitter and its primary interferer provide a pair of transmitters between which the constraint is weak and could be strengthened. These violating test points are sorted into descending order by SIR deficit and for a number of them the constraints between the associated transmitter pairs can be marked for strengthening. The number chosen can vary from one, a small number or even all of them. This can be specified at run time.

If a constraint already exists between a transmitter pair that has been marked for strengthening, the constraint is simply increased by one. If no constraint exists then a new one is inserted. For example if transmitter pairs (1,3) and (2,4) are both marked for strengthening then the constraint file would change as shown below.

```
1   2   >   0                         1   2   >   0
1   3   >   1                         1   3   >   2
1   5   >   0         becomes         1   5   >   0
2   3   >   0                         2   3   >   0
2   5   >   1                         2   4   >   0
3   4   >   0                         2   5   >   1
....                                  3   4   >   0
                                      ....
```

The existing constraint between transmitters 1 and 3 is strengthened from 1 to 2 and a new constraint between transmitters 2 and 4 is created.

This new binary constraint file is passed to the constraint solver FASoft. FASoft produces a new assignment that is again assessed for coverage over a test drid. Violating transmitter pairs are again identified, constraints strengthened and so on until the required coverage is achieved. The coding structure is shown using the pseudo code Figure 4.

```
Input transmitter and receiver coordinates and the required signal to
interference, σ,
Input the number of violating pairs strengthened each time.  (i.e.  1, all,
or a percentage)
Initialise coverage = 0
Initialise strengthen = 0, the number of constraint pairs to be strenghtened
Produce initial binary constraints

WHILE (coverage < 100%) DO

      FOR(i = 1 to strengthen) DO
         IF (constraint already exists between pair i) THEN
            add 1 to the constraint between pair i
         ELSE
            create new co-channel constraint between pair i
         END IF
      END FOR

      Use updated constraint file to produce a new assignment
      using a constraint solver
      Calculate coverage, and cost at each test point
      FOR (i = 1 to the number of reception points) DO
          calculate the signal to interference, SIR, at i
          IF (SIR < σ) THEN
             calculate SIR deficit at i
             identify primary interferer (greatest contributor
             to the violating interference)
             add tuned transmitter and primary interferer to list of
             transmitter pairs that may constraint strengthening
          END IF
      END FOR

      Arrange list of violating pairs by the associated cost
      Calculate strengthen

END WHILE
```

Figure 4: *Automatic constraint strengthening*

2.5 Assignment Evaluation

All assignments are assessed over a *test grid*. This is a 100 by 100 mesh placed over the network with the SIR at each point found and the coverage calculated. Each point on the mesh is assumed to be tuned to the transmitter to which it is nearest and coverage always refers to coverage with respect to the Non-Binary model. Associated pseudo code is given Figure 5.

```
Import N_t transmitter coordinates
Input assignment
Form N_test testpoints - calculate grid point positions
FOR i = 1 to N_test DO

    I_i = 0
    For each testpoint i identify its tuned (nearest) transmitter T_t.
    Calculate the signal strength, S_i at testpoint i

    FOR j = 1 to N_t DO

        IF (T_j ≠ T_t) THEN
           Add the interference at i due to T_j to the current total
           interference, I_i, at i
        END IF

    END FOR

    Calculate the signal to interference ratio S_i/I_i

    IF (S_i/I_i < SIR) THEN
       N_vio = N_vio + 1
       cost = cost + (σ - S_i/I_i)^a
    END IF

END FOR

Calculate and output the percentage coverage given by = 100 × (N_t - N_vio)/N_t
```

Figure 5: *Assignment evaluation*

3 Comparing the Models - Results

3.1 Problem Generator and Reception Points

The Binary and Non-Binary approaches have been tested with the aim of providing coverage for a number of trial networks produced by the problem generator written by Allen and Dunkin [3] . This problem generator places one or more conurbations over a defined region. For each conurbation the factors that influence the population (and therefore transmitter density) are specified and a probability map produced. The problem generator proceeds to place transmitters over the region, with a probability that a transmitter will be at any point given by the above probability map. For example, trial network 1 (two towns, 45 transmitters and 111 receivers) can be seen in Figure 6. Any number of transmitter placements are possible by varying an initial random seed. The problem generator has also been used to locate reception points within the region. These are calculated using so called *Voronoi* polygons surrounding each transmitter. These are polygons such that every point within the polygon is closer to that transmitter then any other (see Figure 7). We assume also, for simplicity, that all points within a polygon are tuned to the transmitter that the polygon surrounds.

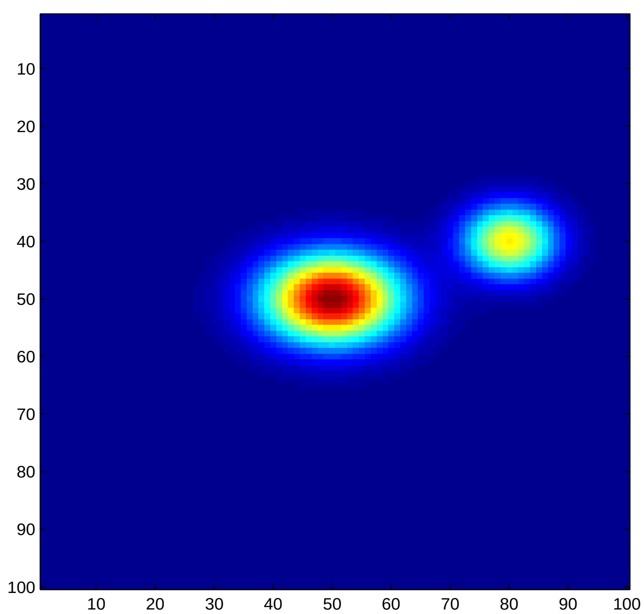

Figure 6: *Two towns, one larger than the other, located on a square region.*

3.2 Reception Points

With the intended aim of providing a specified level of coverage, it would seem sensible to place a fine grid of reception points over the whole region. This would however, involve a great many reception points and would greatly increase the workload of particularly the Non-Binary method in finding assignments. Such a fine grid will thus only be used to evaluate assignments already found. It is necessary then to use a much smaller set of worst case reception points that should be the most likely to experience violating interference. A number of different types of reception points were considered, the best choice being generated by the Voronoi polygons described above. The points of intersection of these polygons are tuned to two or more transmitters and provide appropriate worst case reception points. This is shown diagrammatically for Network 1 in Figure 7.

3.3 Lower Bounds

It should be noted that a lower bound can be generated for the span necessary for 100% coverage. The coverage here can be with respect to any defined set of points, typically either the test grid or the corner points of the Voronoi polygons. This can be done by generating binary constraints using these points as described previously. Since these binary constraints must be satisfied for 100% coverage, it follows that any lower bound (clique bound, spanning tree bound, travelling salesman bound [1] [2] [6]) generated for these binary constraints is a lower bound for 100% coverage at the defined SIR.

3.4 Test Networks

The results of frequency assignments over several test networks, produced by the various Binary and Non-Binary techniques are described in this section. In Figure 8, Figure 9 and Figure 10, assignments at 9dB, 17dB and 25dB for the transmitter and receiver locations within the network described in Figure 1 and Figure 2 are given. The lowest spans for all coverages between 95% and 100% are given for all methods except the greedy sequential algorithm for which it is only currently possible to find coverage at 100%. Although binary constraints are generated from the reception points, the assignments are evaluated over the test grid.

It should be noted that some of the heuristic Non-Binary calculations were only terminated after several days but the Binary calculations took only of the order of several minutes.

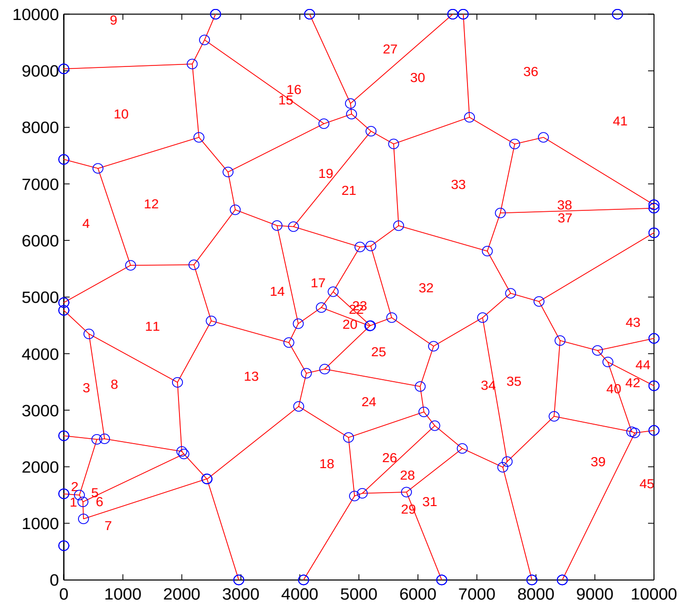

Figure 7: *45 transmitters and 111 receivers were placed as shown. The transmitters are the numbers in the middle of the cells and the receivers are the circles at the cell vertices. Receivers are also placed around the perimeter of the square at all positions equi-distant from two transmitters.*

As an explanation of these results, consider the 17dB case (Figure 9). Binary constraints are calculated at 17dB, used as input to FASoft and the resulting assignment tested over the 100 by 100 test grid. The assignment is found to produce a relatively high 98% coverage over the test grid at a span of 14. To try and achieve a higher coverage at 17dB, the binary constraints are re-calculated at 18dB but the assignment tested at *17dB* over the test grid. Coverage of 99% is achieved, but now at a span of 15. The binary constraints are calculated repeatedly at higher dB until coverage reaches 100% for 17dB over the test grid (to allow comparison with the non-binary results). This is finally achieved by calculating the binary constraints at 21dB at a span of 20. Similarly the binary constraints are calculated at lower dB values to try and attain coverage at less than 98% with a smaller span. This becomes successful at 15dB, with a span of 12 which provides coverage of 95% over the test grid.

Considering the automated selective constraint strengthening, an assignment is produced at an initial SIR that provided coverage at less than 95% for 17dB. The assignment is automatically assessed over the 100 by 100 grid, violating constraints are identified, strengthened and a new assignment produced. The span and coverage gradually increase until coverage at a required level is achieved, for example at 97% with a span of 13.

With the non-binary multiple interferer case, assignments were produced, assessed and

Target SIR = 9dB	SPAN
Binary Span @ 9dB - 1dB = 8dB (Target 95%)	8 for 99% coverage over test grid.
Binary Span @ 9dB - 1dB = 8dB (Target 96%)	8 for 99% coverage over test grid.
Binary Span @ 9dB - 1dB = 8dB (Target 97%)	8 for 99% coverage over test grid.
Binary Span @ 9dB - 1dB = 8dB (Target 98%)	8 for 99% coverage over test grid.
Binary Span @ 9dB - 1dB = 8dB (Target 99%)	8 for 99% coverage over test grid.
Binary Span @ 9dB + 5dB = 14dB (Target 100%)	11 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	7 for 96% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	7 for 96% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	8 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	8 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	8 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	9 for 100% coverage over test grid.
Non-Binary Span @ 9dB (Target 95%)	6 for 97% coverage over test grid.
Non-Binary Span @ 9dB (Target 96%)	6 for 97% coverage over test grid.
Non-Binary Span @ 9dB (Target 97%)	6 for 97% coverage over test grid.
Non-Binary Span @ 9dB (Target 98%)	7 for 98% coverage over test grid.
Non-Binary Span @ 9dB (Target 99%)	8 for 99% coverage over test grid.
Non-Binary Span @ 9dB (Target 100%)	9 for 100% coverage over test grid.
Sequential (greedy) algorithm (Target 100%)	9 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	8
Lower Bound over Test Grid for 100% coverage.	8

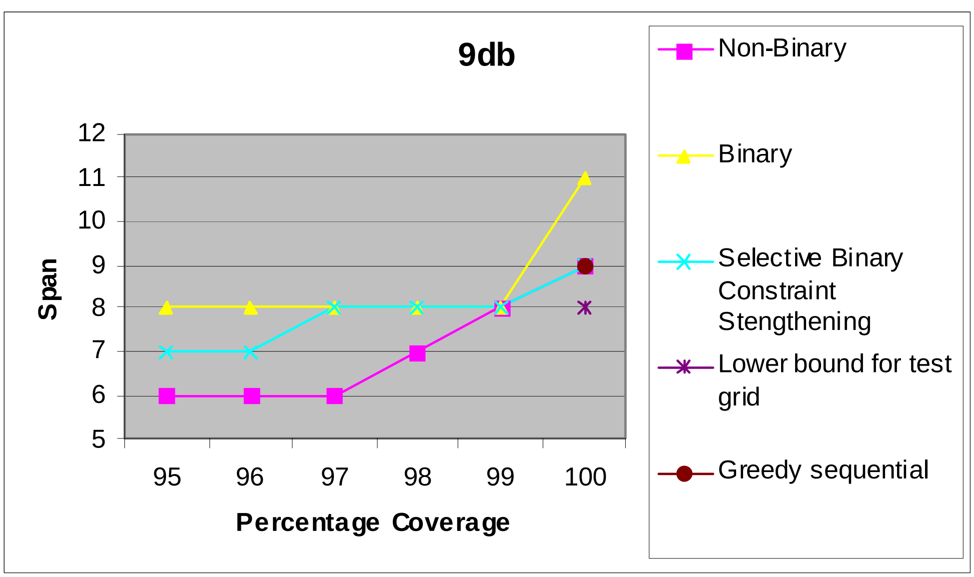

Figure 8: *Results for* $\sigma = 9dB$ *in a network with* $N_t = 45$ *and* $N_r = 111$

Target SIR = 17dB	SPAN
Binary Span @ 17dB - 2dB = 15dB (Target 95%)	12 for 95% coverage over test grid.
Binary Span @ 17dB (Target 96%)	14 for 98% coverage over test grid.
Binary Span @ 17dB (Target 97%)	14 for 98% coverage over test grid.
Binary Span @ 17dB (Target 98%)	14 for 98% coverage over test grid.
Binary Span @ 17dB + 1dB =18dB (Target 99%)	15 for 99% coverage over test grid.
Binary Span @ 17dB + 4dB = 21dB (Target 100%)	20 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	12 for 95% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	13 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	13 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	14 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	15 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	17 for 100% coverage over test grid.
Non-Binary Span @ 17dB (Target 95%)	13 for 96% coverage over test grid.
Non-Binary Span @ 17dB (Target 96%)	13 for 96% coverage over test grid.
Non-Binary Span @ 17dB (Target 97%)	14 for 97% coverage over test grid.
Non-Binary Span @ 17dB (Target 98%)	17 for 98% coverage over test grid.
Non-Binary Span @ 17dB (Target 99%)	19 for 99% coverage over test grid.
Non-Binary Span @ 17dB (Target 100%)	21 for 100% coverage over test grid.
Sequential (greedy) algorithm (Target 100%)	18 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	14
Lower Bound at Test Grid for 100% coverage.	13

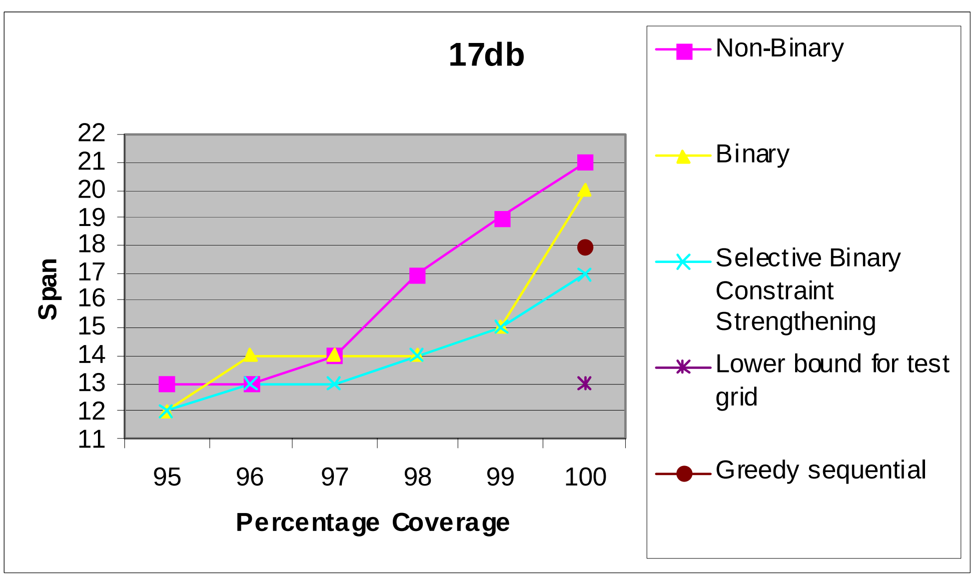

Figure 9: *Results for* $\sigma = 17dB$ *in a network with* $N_t = 45$ *and* $N_r = 111$

Target SIR = 25dB	SPAN
Binary Span @ 25dB - 1dB = 24dB (Target 95%)	24 for 98% coverage over test grid.
Binary Span @ 25dB - 1dB = 24dB (Target 96%)	24 for 98% coverage over test grid.
Binary Span @ 25dB - 1dB = 24dB (Target 97%)	24 for 98% coverage over test grid.
Binary Span @ 25dB (Target 98%)	26 for 99% coverage over test grid.
Binary Span @ 25dB (Target 99%)	26 for 99% coverage over test grid.
Binary Span @ 25dB - 4dB = 29dB (Target 100%)	34 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	23 for 96% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	23 for 96% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	24 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	26 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	26 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	32 for 100% coverage over test grid.
Non-Binary Span @ 25dB (Target 95%)	26 for 95% coverage over test grid.
Non-Binary Span @ 25dB (Target 96%)	28 for 97% coverage over test grid.
Non-Binary Span @ 25dB (Target 97%)	28 for 97% coverage over test grid.
Non-Binary Span @ 25dB (Target 98%)	29 for 98% coverage over test grid.
Non-Binary Span @ 25dB (Target 99%)	30 for 99% coverage over test grid.
Non-Binary Span @ 25dB (Target 100%)	39 for 100% coverage over test grid.
Sequential (greedy) algorithm (Target 100%)	32 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	26
Lower Bound at Test Grid for 100% coverage.	23

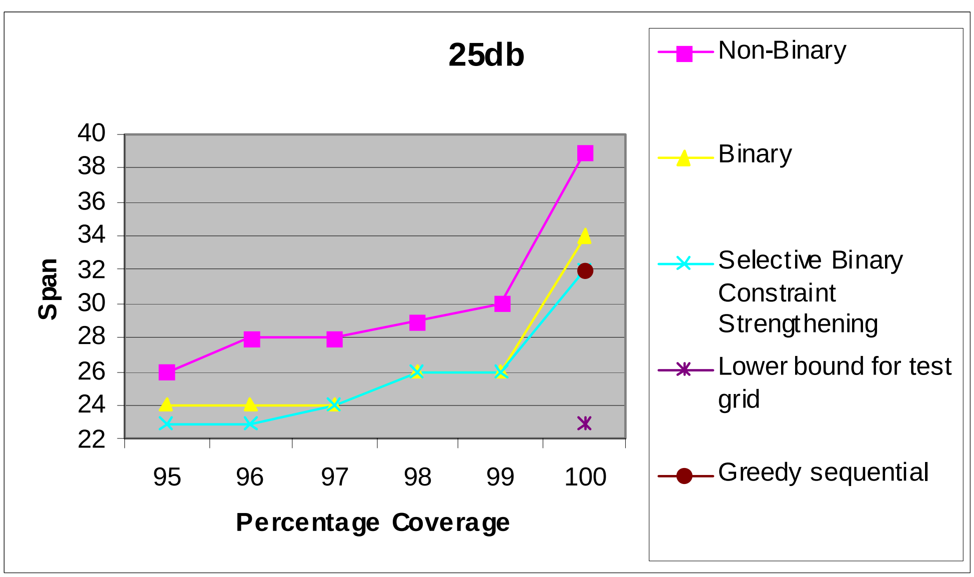

Figure 10: *Results for* $\sigma = 25dB$ *in a network* $N_t = 45$ *and* $N_r = 111$

altered on the basis of coverage over the reception points. Periodically the best assignment so far over the *reception* points was assessed for coverage over the set of *test* points and if it was the best so far at a particular required coverage then the assignment was recorded. It was possible to allow the required coverage over the reception points to drop well below the 95% minimum required by the test points. This can be seen for both the binary and the non-binary methods where the coverage over the reception points is always no greater than or indeed a great deal less than the coverage over the test points. Indeed in some instances the assignment that produces 100% coverage over the test points produces less than this over the reception points. This suggests that during these runs the test grid is not fine enough or at least that the reception points should be added to the points that compose the evaluation test grid.

The sequential algorithm currently implemented can only find assignments that produce 100% coverage. The span given, 18 in Figure 9, is the lowest achieved after a reasonable time (up to 2 days).

A typical signal/interference map of the region and also a cost map that shows the positions of the violating test points at 97% coverage are shown in Figure 11 adn Figure 12 below.

Regarding 100% coverage, the sequential method appears far more successful than the heuristic method and achieves its results in a fraction of the time. The heuristic method also employed a sequential proceedure to provide the first assignment. It may be worth investigating the replacement of the single sequential proceedure used to initialise the non-binary heuristic by the sequential algorithm described in section 2.

From the tables within Figure 8, Figure 9 and Figure 10, it appears clear that the situation changes as the required SIR, σ, increases. For 9dB, within the time allotted the non-binary, heuristic method is clearly more successful. However as σ increases, the method's performance degrades whereas the Binary methods performs uniformly for all σ. Both Binary methods become more successful than the Non-Binary at 17dB and much more so at 25dB. The degradation of the performance of the Non-Binary method with increasing σ is probably caused by the size of the frequency domain. The higher the SIR, the higher the required span and the higher the number of possible assignments, hence a reduced chance of finding an adequate assignment at each iteration. With network 1 with 45 transmitters, at 9dB the Non-Binary span is 9 for 100% coverage and there are 9^{45} possible assignments. At 25dB the Non-Binary span is 39 for 100% coverage and there are 39^{45} possible assignments. Hence there are $(39/9)^{45} = 4.5 \times 10^{28}$ times as many possible trial assignments at 25dB as there are at 9dB. Hence it is computationally more difficult to find a good assignment at these higher values of σ.

The same general argument applies with problem size. The more transmitters in the

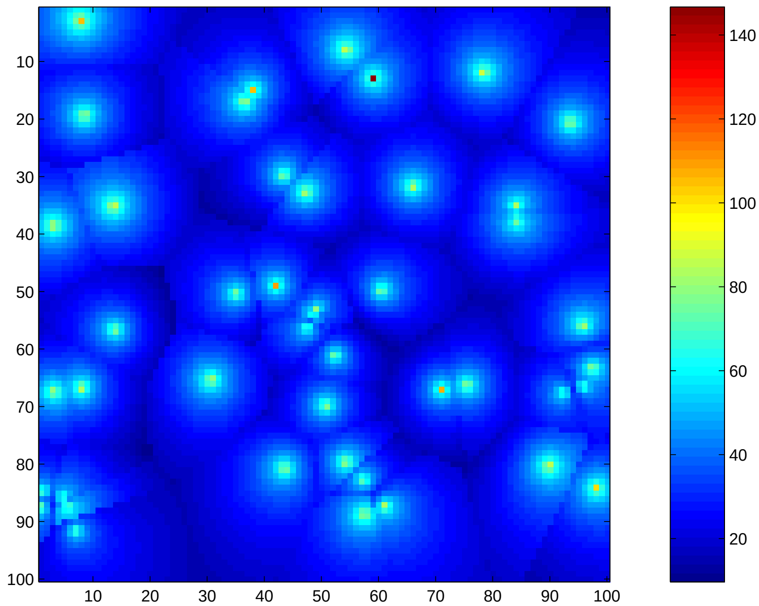

Figure 11: *Map of the Signal to Interference ratio over the region of Network 1 when the Coverage provided is 97% at 17dB. The colour-bar shows the SIR corresponding to each colour.*

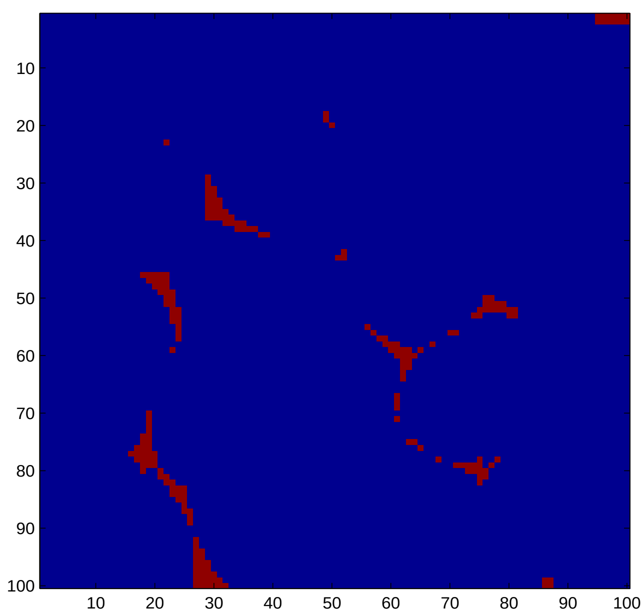

Figure 12: *Map showing the values of the cost function E over the region of Network 1 when the coverage provided is 97% at 17dB. The paler patches of non-zero cost correspond to some of the darker patches of Figure 11.*

network, the more potential trial assignments. Subsequently you would expect the overall performance of the Non-Binary method to worsen with increased network size. This can be clearly seen in the range of problems discussed below.

Figure 13 and Figure 14 and also Figure 15 and Figure 16 show the results at 9dB and 17dB for smaller networks (15 and 27 transmitters) than the network shown in Figure ??. The Non-Binary heuristic method does relatively well, although its span at 100% is matched by the performance of the sequential method and the automated individual constraint strengthening procedure. Considering a slightly larger problem (95 transmitters) at 9dB and 17dB (Figure 17 and Figure 18), the distinction between the performance of the binary and the non-binary, heuristic methods becomes more obvious, particularly at 17dB. The sequential method continues to do well but the most striking feature in this example is the relative success of the automated individual constraint strengthening procedure over simply calculating constraints at various distinct SIRs. Clearly the constraint solver is able to convert the increased subtlety used in choosing which constraints to strengthen into an improvement in the actual assignment produced.

Moving on to a much larger, realistically sized problem of 458 transmitters, (the results for 9dB and 17dB are shown in Figure 19 and Figure 20), both the non-binary heuristic and sequential approaches do not produce competitive results within a set time and therefore are not shown. The individual constraint strengthening approach offers a clear improvement on the standard binary method, particularly at 100% where its span is improved by over 50%.

Target SIR = 9dB	SPAN
Binary Span @ 9dB - 1dB = 8dB (Target 98%)	8 for 98% coverage over test grid.
Binary Span @ 9dB - 1dB = 8dB (Target 98%)	8 for 98% coverage over test grid.
Binary Span @ 9dB - 1dB = 8dB (Target 98%)	8 for 98% coverage over test grid.
Binary Span @ 9dB - 1dB = 8dB (Target 98%)	8 for 98% coverage over test grid.
Binary Span @ 9dB (Target 99%)	9 for 99% coverage over test grid.
Binary Span @ 9dB + 2dB = 11dB (Target 100%)	10 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	7 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	7 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	7 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	7 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	8 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	9 for 100% coverage over test grid.
Non-Binary Span @ 9dB (Target 95%)	5 for 95% coverage over test grid.
Non-Binary Span @ 9dB (Target 96%)	6 for 98% coverage over test grid.
Non-Binary Span @ 9dB (Target 97%)	6 for 98% coverage over test grid.
Non-Binary Span @ 9dB (Target 98%)	6 for 98% coverage over test grid.
Non-Binary Span @ 9dB (Target 99%)	7 for 99% coverage over test grid.
Non-Binary Span @ 9dB (Target 100%)	9 for 100% coverage over test grid.
Sequential (greedy) algorithm (Target 100%)	17 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	9
Lower Bound at Test Grid for 100% coverage.	9

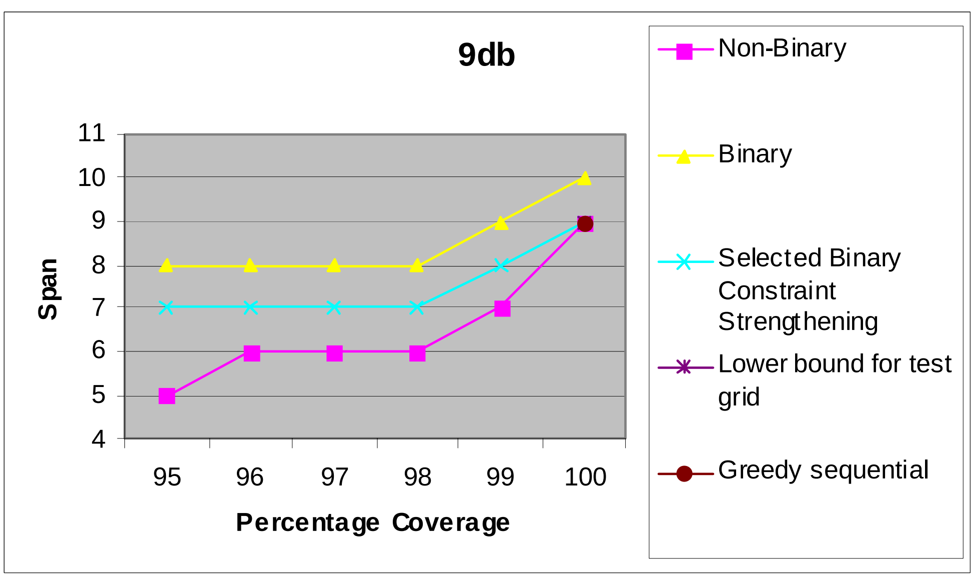

Figure 13: *Results for* $\sigma = 9dB$ *in a network with* $N_t = 15$ *and* $N_r = 40$

Target SIR = 17dB	SPAN
Binary Span @ 17dB - 2dB = 15dB (Target 98%)	10 for 98% coverage over test grid.
Binary Span @ 17dB - 2dB = 15dB (Target 98%)	10 for 98% coverage over test grid.
Binary Span @ 17dB - 2dB = 15dB (Target 98%)	10 for 98% coverage over test grid.
Binary Span @ 17dB - 2dB = 15dB (Target 98%)	10 for 98% coverage over test grid.
Binary Span @ 17dB (Target 99%)	11 for 99% coverage over test grid.
Binary Span @ 17dB + 3dB = 20dB (Target 100%)	16 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	10 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	10 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	10 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	10 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	11 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	12 for 100% coverage over test grid.
Non-Binary Span @ 17dB (Target 95%)	9 for 97% coverage over test grid.
Non-Binary Span @ 17dB (Target 96%)	9 for 97% coverage over test grid.
Non-Binary Span @ 17dB (Target 97%)	9 for 97% coverage over test grid.
Non-Binary Span @ 17dB (Target 98%)	10 for 99% coverage over test grid.
Non-Binary Span @ 17dB (Target 99%)	10 for 99% coverage over test grid.
Non-Binary Span @ 17dB (Target 100%)	12 for 100% coverage over test grid.
Sequential (greedy) algorithm (Target 100%)	12 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	11
Lower Bound at Test Grid for 100% coverage.	11

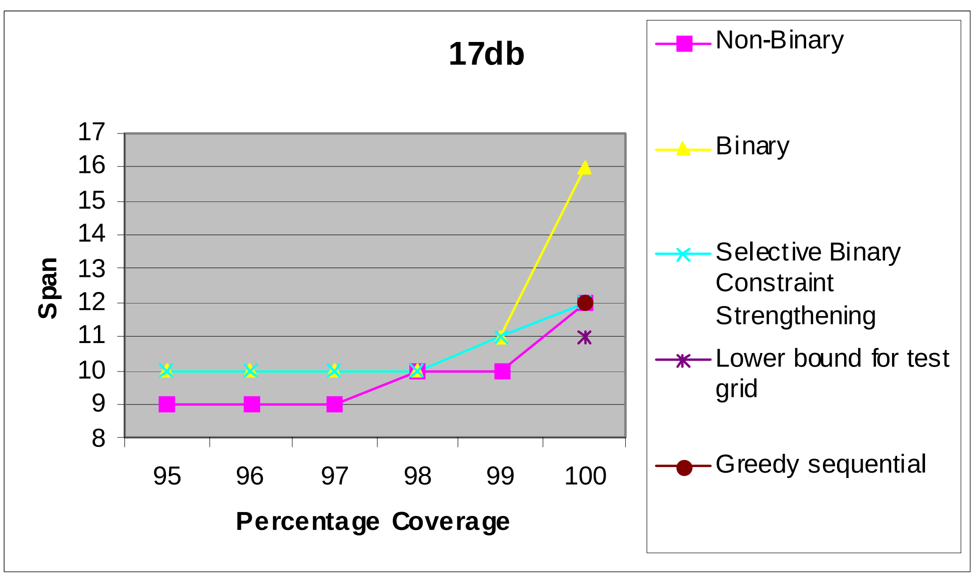

Figure 14: *Results for* $\sigma = 17dB$ *in a network with* $N_t = 15$ *and* $N_r = 111$

Target SIR = 9dB	SPAN
Binary Span @ 9dB - 1dB =8dB (Target 97%)	6 for 97% coverage over test grid.
Binary Span @ 9dB - 1dB =8dB (Target 97%)	6 for 97% coverage over test grid.
Binary Span @ 9dB - 1dB =8dB (Target 97%)	6 for 97% coverage over test grid.
Binary Span @ 9dB + 1dB =10dB (Target 99%)	7 for 99% coverage over test grid.
Binary Span @ 9dB + 1dB =10dB (Target 99%)	7 for 99% coverage over test grid.
Binary Span @ 9dB + 2dB = 11dB (Target 100%)	9 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	5 for 95% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	5 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	5 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	5 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	6 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	8 for 100% coverage over test grid.
Non-Binary Span @ 9dB (Target 95%)	6 for 96% coverage over test grid.
Non-Binary Span @ 9dB (Target 96%)	6 for 96% coverage over test grid.
Non-Binary Span @ 9dB (Target 97%)	6 for 97% coverage over test grid.
Non-Binary Span @ 9dB (Target 98%)	6 for 98% coverage over test grid.
Non-Binary Span @ 9dB (Target 99%)	8 for 99% coverage over test grid.
Non-Binary Span @ 9dB (Target 100%)	8 for 100% coverage over test grid.
Sequential (greedy) algorithm (Target 100%)	8 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	7
Lower Bound at Test Grid for 100% coverage.	7

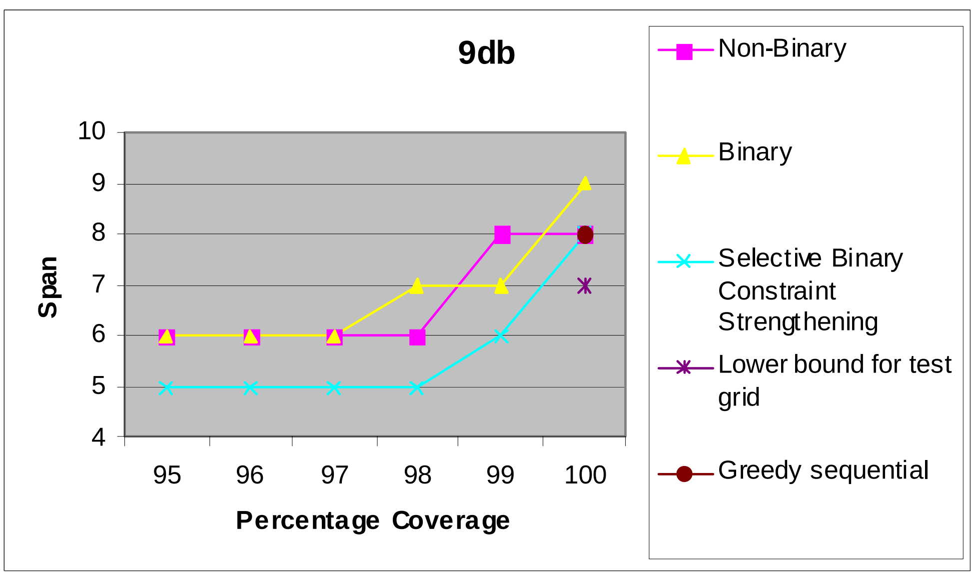

Figure 15: *Results for* $\sigma = 9dB$ *in a network with* $N_t = 27$ *and* $N_r = 68$

Target SIR = 17dB	SPAN
Binary Span @ 17dB + 1dB =18dB (Target 99%)	13 for 99% coverage over test grid.
Binary Span @ 17dB + 1dB =18dB (Target 99%)	13 for 99% coverage over test grid.
Binary Span @ 17dB + 1dB =18dB (Target 99%)	13 for 99% coverage over test grid.
Binary Span @ 17dB + 1dB =18dB (Target 99%)	13 for 99% coverage over test grid.
Binary Span @ 17dB + 1dB =18dB (Target 99%)	13 for 99% coverage over test grid.
Binary Span @ 17dB + 3dB = 20dB (Target 100%)	17 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	12 for 96% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	12 for 96% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	13 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	13 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	13 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	15 for 100% coverage over test grid.
Non-Binary Span @ 9dB (Target 95%)	14 for 99% coverage over test grid.
Non-Binary Span @ 9dB (Target 96%)	14 for 99% coverage over test grid.
Non-Binary Span @ 9dB (Target 97%)	14 for 99% coverage over test grid.
Non-Binary Span @ 9dB (Target 98%)	14 for 99% coverage over test grid.
Non-Binary Span @ 9dB (Target 99%)	14 for 99% coverage over test grid.
Non-Binary Span @ 9dB (Target 100%)	16 for 100% coverage over test grid.
Sequential (greedy) algorithm (Target 100%)	15 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	13
Lower Bound at Test Grid for 100% coverage.	13

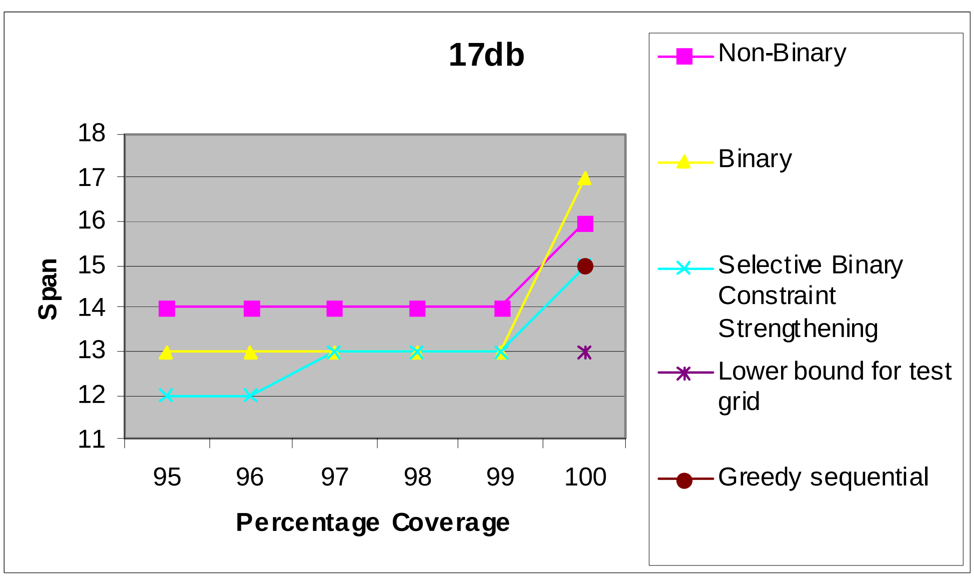

Figure 16: *Results for* $\sigma = 17dB$ *in a network with* $N_t = 27$ *and* $N_r = 68$

Target SIR = 9dB	SPAN
Binary Span @ 9dB + 2dB = 11dB (Target 97%)	15 for 97% coverage over test grid.
Binary Span @ 9dB + 2dB = 11dB (Target 97%)	15 for 97% coverage over test grid.
Binary Span @ 9dB + 2dB = 11dB (Target 97%)	15 for 97% coverage over test grid.
Binary Span @ 9dB + 3dB = 12dB (Target 99%)	16 for 99% coverage over test grid.
Binary Span @ 9dB + 3dB = 12dB (Target 99%)	16 for 99% coverage over test grid.
Binary Span @ 9dB + 6dB = 15dB (Target 100%)	21 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	11 for 95% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	12 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	13 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	13 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	13 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	16 for 100% coverage over test grid.
Non-Binary Span @ 9dB (Target 95%)	13 for 96% coverage over test grid.
Non-Binary Span @ 9dB (Target 96%)	13 for 96% coverage over test grid.
Non-Binary Span @ 9dB (Target 97%)	14 for 97% coverage over test grid.
Non-Binary Span @ 9dB (Target 98%)	15 for 98% coverage over test grid.
Non-Binary Span @ 9dB (Target 99%)	19 for 99% coverage over test grid.
Non-Binary Span @ 9dB (Target 100%)	22 for 100% coverage over test grid.
Sequential (greedy) algorithm (Target 100%)	17 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	11
Lower Bound at Test Grid for 100% coverage.	9

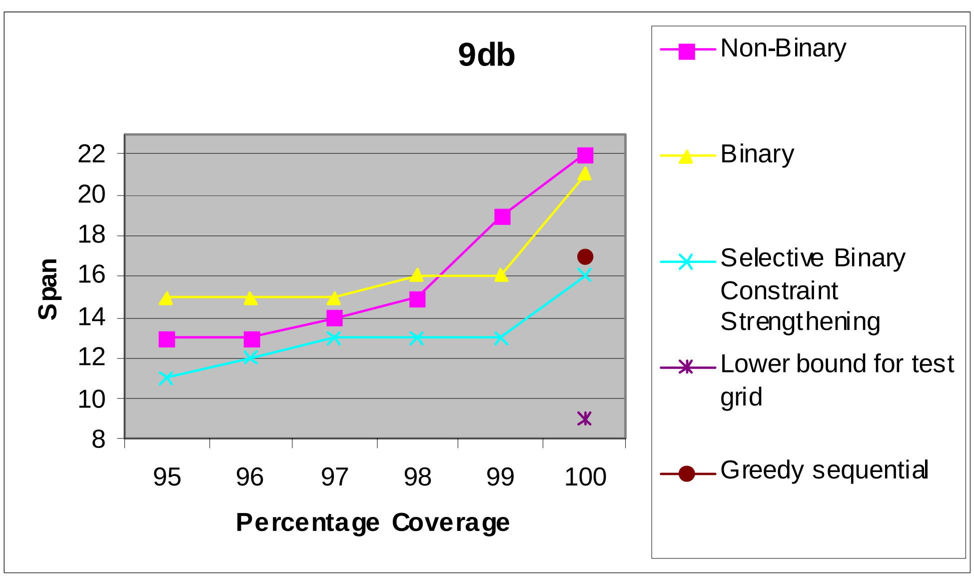

Figure 17: *Results for* $\sigma = 9dB$ *in a network with* $N_t = 95$ *and* $N_r = 210$

Target SIR = 17dB	SPAN
Binary Span @ 17dB - 2dB = 15dB (Target 95%)	25 for 95% coverage over test grid.
Binary Span @ 17dB (Target 96%)	29 for 98% coverage over test grid.
Binary Span @ 17dB (Target 97%)	29 for 98% coverage over test grid.
Binary Span @ 17dB (Target 98%)	29 for 98% coverage over test grid.
Binary Span @ 17dB + 1dB =18dB (Target 99%)	31 for 99% coverage over test grid.
Binary Span @ 17dB + 4dB = 21dB (Target 100%)	38 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	23 for 95% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	24 for 96% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	25 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	25 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	27 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	35 for 100% coverage over test grid.
Non-Binary Span @ 17dB (Target 95%)	28 for 96% coverage over test grid.
Non-Binary Span @ 17dB (Target 96%)	35 for 97% coverage over test grid.
Non-Binary Span @ 17dB (Target 97%)	35 for 97% coverage over test grid.
Non-Binary Span @ 17dB (Target 98%)	37 for 98% coverage over test grid.
Non-Binary Span @ 17dB (Target 99%)	40 for 99% coverage over test grid.
Non-Binary Span @ 17dB (Target 100%)	55 for 100% coverage over test grid.
Sequential (greedy) algorithm (Target 100%)	37 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	25
Lower Bound at Test Grid for 100% coverage.	20

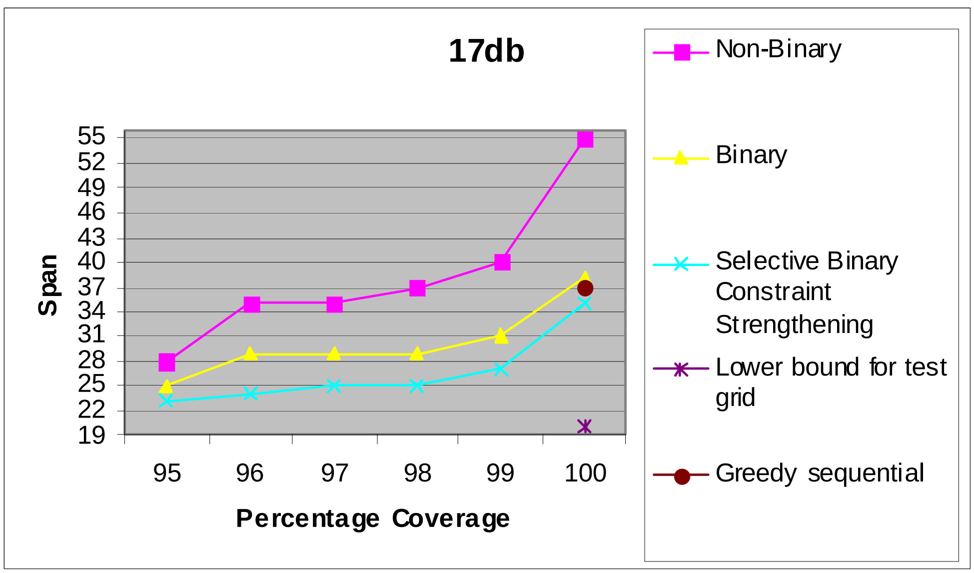

Figure 18: *Results for* $\sigma = 17dB$ *in a network with* $N_t = 95$ *and* $N_r = 210$

Target SIR = 9dB	SPAN
Binary Span @ 9dB - 1dB = 8dB (Target 95%)	9 for 96% coverage over test grid.
Binary Span @ 9dB (Target 96%)	10 for 97% coverage over test grid.
Binary Span @ 9dB (Target 97%)	10 for 97% coverage over test grid.
Binary Span @ 9dB + 4dB = 13dB (Target 98%)	11 for 99% coverage over test grid.
Binary Span @ 9dB + 4dB = 13dB (Target 99%)	11 for 99% coverage over test grid.
Binary Span @ 9dB + 8dB = 17dB (Target 100%)	19 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	9 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	9 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	9 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	10 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	11 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	13 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	11
Lower Bound at Test Grid for 100% coverage.	8

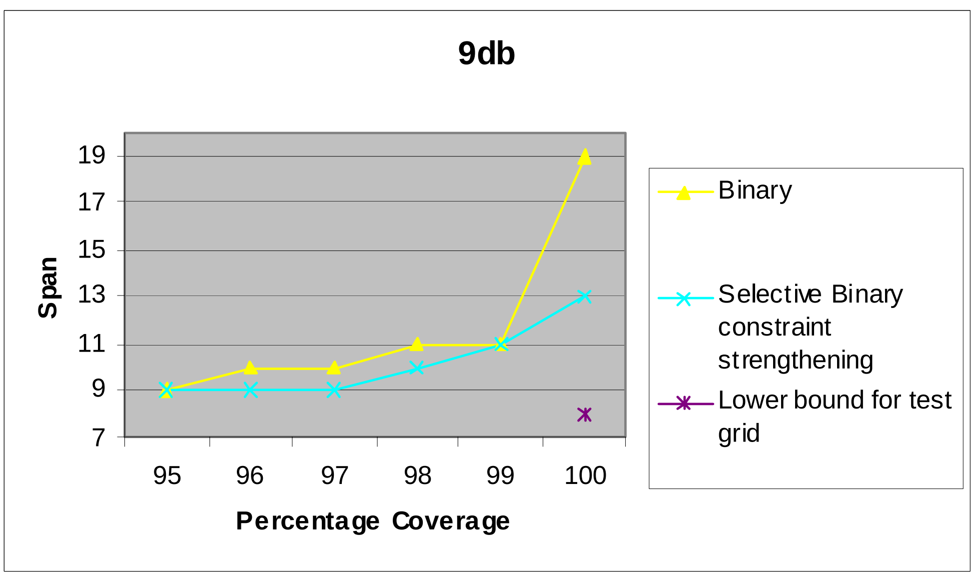

Figure 19: *Results for* $\sigma = 9dB$ *in a network with* $N_t = 458$ *and* $N_r = 976$

Target SIR = 17dB	SPAN
Binary Span @ 17dB + 1dB = 18dB (Target 96%)	20 for 96% coverage over test grid.
Binary Span @ 17dB + 1dB = 18dB (Target 96%)	20 for 96% coverage over test grid.
Binary Span @ 17dB + 2dB = 19dB (Target 97%)	22 for 97% coverage over test grid.
Binary Span @ 17dB + 3dB = 20dB (Target 98%)	25 for 98% coverage over test grid.
Binary Span @ 17dB + 4dB = 21dB (Target 99%)	27 for 99% coverage over test grid.
Binary Span @ 17dB + 9dB = 26dB (Target 100%)	43 for 100% coverage over test grid.
Automatic Constraint Strengthening (Target 95%)	19 for 95% coverage over test grid.
Automatic Constraint Strengthening (Target 96%)	20 for 96% coverage over test grid.
Automatic Constraint Strengthening (Target 97%)	21 for 97% coverage over test grid.
Automatic Constraint Strengthening (Target 98%)	23 for 98% coverage over test grid.
Automatic Constraint Strengthening (Target 99%)	25 for 99% coverage over test grid.
Automatic Constraint Strengthening (Target 100%)	28 for 100% coverage over test grid.
Lower Bound at Reception Points for 100% coverage.	19
Lower Bound at Test Grid for 100% coverage.	14

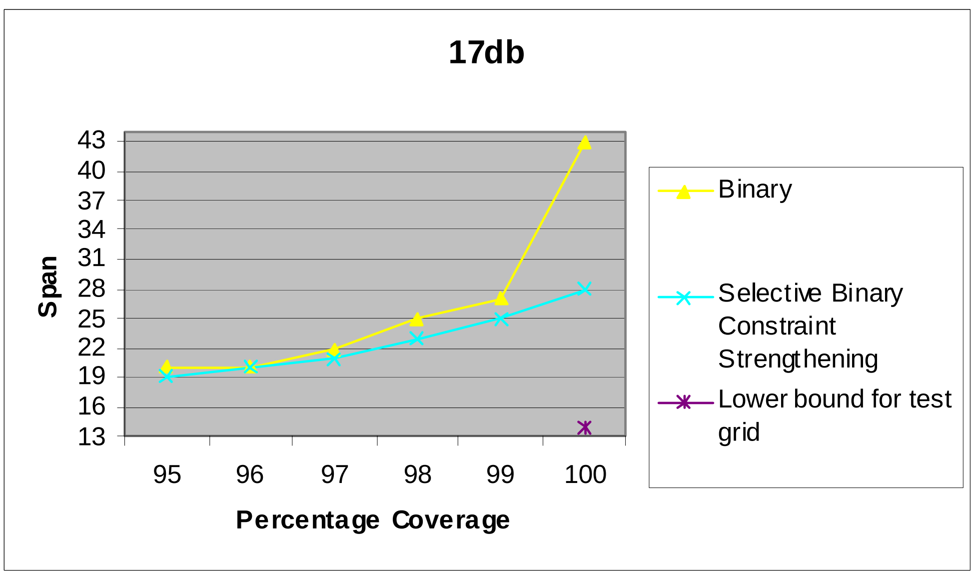

Figure 20: *Results for* $\sigma = 17dB$ *in a network with* $N_t = 458$ *and* $N_r = 976$

3.5 Network Classification

The automated constraint strengthening routine was used to begin an investigation into the potential for characterising the span of networks in terms of a number of parameters. This could enable approximate prediction of what span and therefore the total frequency allocation that may be necessary to satisfy the required coverage over such a network.

The problem generator produces transmitter and receiver co-ordinates over a probability map that simulates the area around one or more conurbations. The probability at a point is proportional to the traffic and therefore to the likelihood of a transmitter being located there. A single probability spike represents a single conurbation and is defined by its *height* (height of the central peak) and its *width* which dictates the greatest distance from the peak at which the probability is greater than the background probability. A single conurbation, placed centrally within an area and parameterised by its height and width was used to initially test span prediction. These parameters are essentially internal parameters of the problem generator used. Eventually they will be more closely related to real world parameters.

An algorithmic loop was set up that formed networks at a variety of heights and widths. For each probability map, 25 networks were formed and spans for 100% coverage at 9dB were found. Typical variations in the span for a number of width values at a particular height are shown in Figure 21. The average span for each height and width was found and is shown in Figure 22. The average spans are then passed with the heights and widths to a statistical routine that used multiple regression to produce the equation given below that relates the span to the height and width.

$$Span = 43.96 \times Height + 226.52 \times Width + 16.42$$

Additional statistics produced by the routine showed that the equation was 97% accurate over the range of the data.

As a test, a range of networks constructed at 9dB and based on a height of 0.067 and a width of 0.005 (not used in construction of the equation) were constructed and the average span was found to be exactly 21 (max 24, min 17). Using the above formula we obtain the predicted span as follows

$$\begin{aligned} Span &= 43.96 \times 0.067 + 226.52 \times 0.005 + 16.42 \\ &= 20.498 \end{aligned}$$

This solution is within 3% of the actual average span.

When more data was added from networks constructed at 17dB, it was possible to construct a formula for span that incorporated SIR as well as the height and width. The new formula is:

$$Span = 77.9 \times Height + 498.42 \times Width + 3.62 \times SIR - 22.09$$

Using this formula with the previous example we obtain the predicted span as

$$\begin{aligned} Span &= 77.9 \times 0.067 + 498.4 \times 0.005 + +3.62 \times 9 - 22.09 \\ &= 18.2 \end{aligned}$$

Although this is not as close as the first formula, it is still within the bounds formed by the maximum and the minimum. This formula would clearly in inaccurate for small values of SIR.

Another example at 17dB, with a height of 0.085 and a width of 0.0095 had an average span of 51.52 (max 58, min 43)

The prediction formula gives a value for the span as

$$\begin{aligned} Span &= 77.9 \times 0.085 + 498.4 \times 0.0095 + +3.62 \times 17 - 22.09 \\ &= 50.8 \end{aligned}$$

This is within 2% of the actual average span.

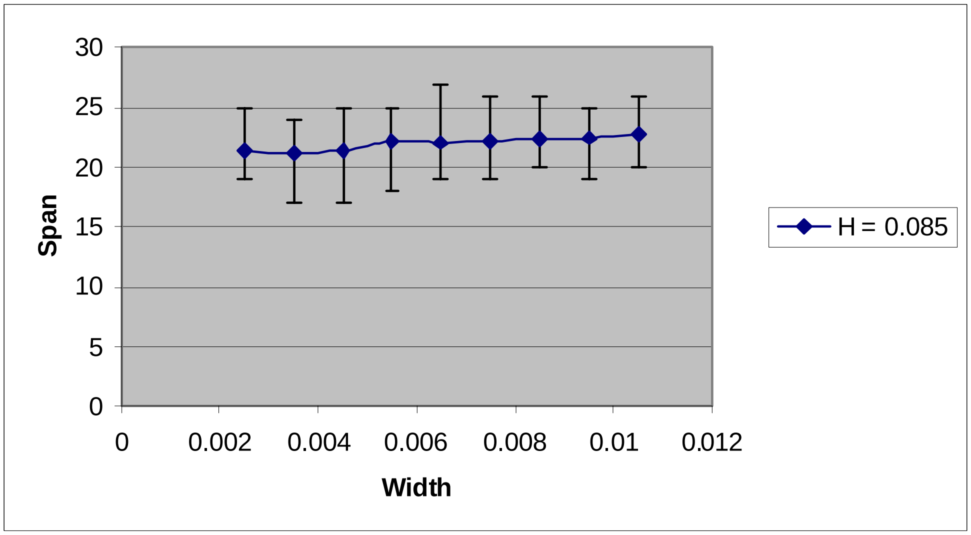

Figure 21: *Variation of span over 25 iterations for a number of width values at a given height.*

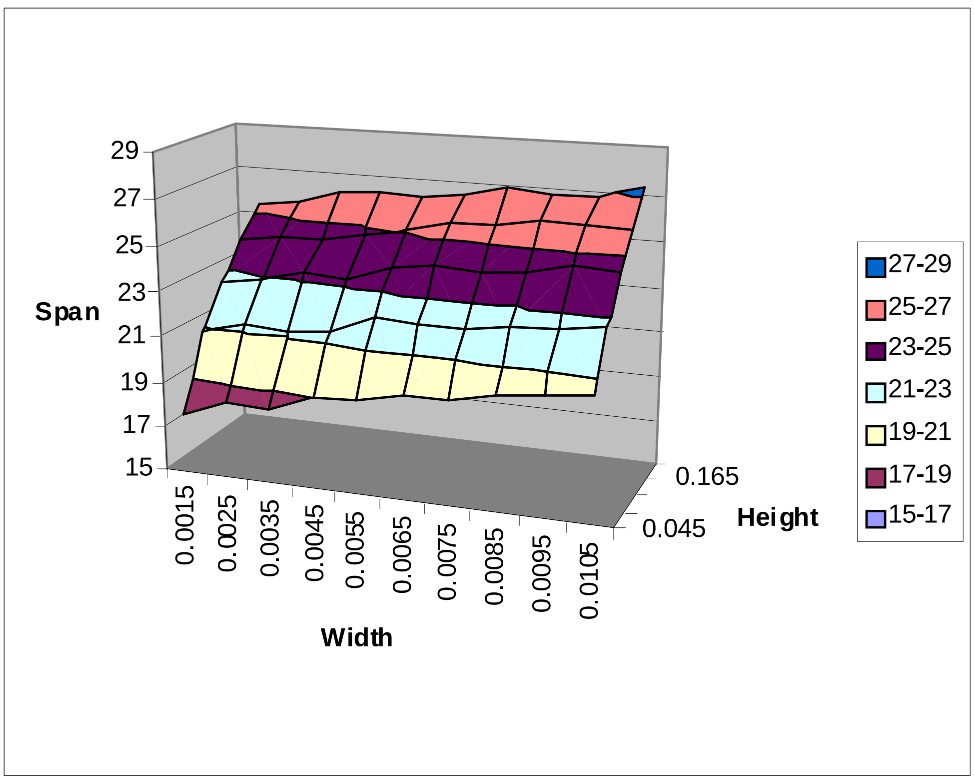

Figure 22: *Variation of the average span for 100% coverage at 9dB over a range of height and width values.*

4 Conclusions and Future Work

Although for small problems and at a low SIR the non-binary heuristic method may equal the other methods, it is clear that for larger problems and a larger SIR it is not competitive with either of the binary models or, in the case of 100% coverage, with the sequential method. The method may still prove useful eventually in making final improvements to an initial good assignment found by faster methods.

The success of the sequential method at 100% coverage was slightly unexpected. It may be useful to use a hybrid of this method with the above non-binary heuristic. The sequential method itself may be improved as follows:

1. A heuristic could be applied to the sequential algorithm to control the number of transmitters reordered at each iteration. It may turn out to be beneficial to start by reordering relatively large numbers of transmitters, but then reduce the number over time in some way. Meta heuristics would seem suitable for this purpose.

2. The sequential algorithm is currently slow for large problems, but this may be partially overcome using binary constraints. Some possible values for the frequencies will violate one or more constraints and therefore the time consuming signal-to-interference calculation will not have to be performed at that frequency.

3. The sequential method may also be adapted for less than 100% coverage in a way analogous to the original binary method. If 95% coverage is required at 9dB for example, then determining an assignment based on several smaller values of the SIR may produce 95% coverage at 9dB.

The approach of selectively strengthening binary constraints is fairly successful. However, as not all of the interference information is used, there will be instances where the constraints *cannot* find the best allocation. In these cases it may be advantageous to use non-binary constraints in specific areas where the assignment is difficult. This would add to the run time of the problem, but the introduction of more complete information may aid the assignment. The method may also be made more effective by allowing constraint weakening as well as constraint strengthening, if a suitable heuristic can be devised.

The incorporation of non-binary constraints may also allow improvement of the lower bounding technique. A method of doing this using non-binary constraints is under consideration currently. It may be that closing the gap between the best span obtained and the best lower bound requires a bigger improvement to the lower bound than to the assignment technique.

Currently it is assumed that the simple inverse fourth power propagation law is adequate for comparing methods and algorithms. More realistic results could be obtained by replacing the simple inverse fourth power law by a more accurate propagation loss algorithm. This would both demonstrate that the conclusions on the relative merits of different algorithms remain valid, and make the algorithms more practically useful.

The multiple regression based technique for predicting the span of a network in terms of the parameters will need additional development as the results improve. It is probable that more parameters will be required, and that these should more closely describe the real world than the internal workings of the problem generator.

5 Dissemination and Publication

The contents of this report will be discussed informally, at a meeting at Gatwick in January 1999. Parts of the report were presented at "INFORMS 98", a major OR/IT conference held in Tel Aviv, Israel in June 1998. In September 1998, the work was presented at a NATO RTA conference in Aalborg, Denmark. A paper including part of this work will appear in the conference proceedings. It is anticipated that further papers will be written in the near future.

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